

Interest Rate Derivatives

Fixed Income Trading Strategies

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Please note

The definitions of basis and cost of carry have been changed in this version of the brochure.

In the previous version, the following definitions were used:

$\text{Basis} = \text{Futures Price} - \text{Price of Cash Instrument}$

$\text{Cost of Carry} = \text{Basis}$

In this version, the following definitions are used:

$\text{Basis} = \text{Price of Cash Instrument} - \text{Futures Price}$

$\text{Cost of Carry} = \text{Basis}$

These changes have been made in order to ensure that definitions of both items are consistent throughout Eurex materials, including the Trader Examination and corresponding preparatory materials.

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Brochure Structure and Objectives

This brochure describes the fixed income derivatives traded at Eurex and illustrates some of their most significant applications. These contracts are comprised of futures on fixed income securities ("fixed income futures") and options on fixed income futures. To provide a better understanding of the contracts described, the fundamental characteristics of fixed income securities and the indicators used to analyze them will be outlined. Basic knowledge of the securities industry is a prerequisite. Explanations of fixed income securities contained in this brochure predominantly refer to such issues on which Eurex fixed income derivatives are based.

Characteristics of Fixed Income Securities

Bonds – Definition

A bond can be described as large-scale borrowing on the capital market, whereby the creditor's entitlements are certified in the form of securities. The offerings of securities are known as issues and the respective debtor as the issuer. Bonds are categorized according to their lifetime, issuer, interest payment details, credit rating and other factors. Fixed income bonds bear a fixed interest payment, known as the coupon, which is based on the nominal value of the bond. Depending on the specifications, interest payment is usually semi-annual or annual. Fixed income derivatives traded at Eurex are based on a basket of either German or Swiss fixed income public sector bonds.

In Switzerland, the Swiss National Bank (SNB) manages the borrowing requirements for the Swiss Federal Finance Administration. Capital is raised by issuing so-called "money market book-entry claims" as well as Treasury Notes and Confederation Bonds. Only the Confederation Bonds with different lifetimes are freely tradable. Other government bonds are exchanged only between the SNB and banks, or in interbank trading.

The German Finance Agency (Bundesrepublik Deutschland – Finanzagentur GmbH) has been responsible for issuing German Government Bonds (on behalf of the German Government) since June 2001. Other publicly tradable issues include bonds issued up until 1995 by the former "Treuhandanstalt" privatization agency and the German Federal Government's special funds, for example, the "German Unity Fund". These bonds are attributed the same creditworthiness as a result of the assumption of liability by the Federal Republic of Germany. German government issues relevant to the Eurex fixed income derivatives have the following lifetimes and coupon payment details.

Government issues	Lifetime	Coupon payment
German Federal Treasury Notes (Bundesschatzanweisungen)	2 years	Annual
German Federal Debt Obligations (Bundessobligationen)	5 years	Annual
German Government Bonds (Bundesanleihen)	10 and 30 years	Annual

The terms of these issues do not provide for early redemption by calling in or drawing¹.

¹ Cf. Deutsche Bundesbank, "Der Markt für deutsche Bundeswertpapiere" (The German Government securities market), 2nd edition, Frankfurt/Main, 1998.

In this chapter the following information will be used for a number of explanations and calculations:

Example:

Debt security issue	German Government Bond
... by the issuer	Federal Republic of Germany
... at the issue date	July 5, 2001
... with a lifetime of	10 years
... a redemption date on	July 4, 2011
... a fixed interest rate of	4.5%
... coupon payment	annual
... a nominal value of	100

Lifetime and Remaining Lifetime

One must differentiate between lifetime and remaining lifetime in order to understand fixed income bonds and related derivatives. The lifetime denotes the time period from the time of issue until the nominal value of the security is redeemed, while the remaining lifetime is the remaining time period from the valuation date until redemption of securities already issued.

Example:

The bond has a lifetime of	10 years
... as at the valuation date	March 11, 2002 ("today")
... the remaining lifetime is	9 years and 115 days

Nominal and Actual Rate of Interest (Coupon and Yield)

The nominal interest rate of a fixed income bond is the value of the coupon relative to the nominal value of the security. In general, neither the issue price nor the traded price of a bond corresponds to its nominal value. Instead, bonds are traded below or above par; i.e. their value is below or above the nominal value of 100 percent. Both the coupon payments and the actual capital invested are taken into account when calculating the yield. This means that, unless the bond is traded at exactly 100 percent, the actual rate of interest – in other words: the yield – deviates from the nominal rate of interest. The actual rate of interest is lower (higher) than the nominal rate of interest for a bond trading above (below) its nominal value.

Example:

The bond has	
... a nominal value of	100
... but is trading at a price of	102.50
... a fixed interest rate of	4.5%
... a coupon of	$4.5\% \times 100 = 4.50$
... a yield of	4.17% ²

In this case the bond's yield is lower than the nominal rate of interest.

Accrued Interest

When a bond is issued, it may be subsequently bought and sold many times in between the predetermined future coupon anniversary dates. As such the buyer pays the seller the interest accrued up to the value date of the transaction, as he/she will receive the full coupon at the next coupon payment date. The interest accrued from the last coupon payment date up to the valuation date is referred to as the accrued interest.

Example:

The bond is purchased on	March 11, 2002 ("today")
The interest is paid	annually, on July 4
The coupon rate is	4.5%
The time period since the last coupon payment is	250 days ³
This results in accrued interest of	$4.5\% \times 250 / 365 = 3.08\%$

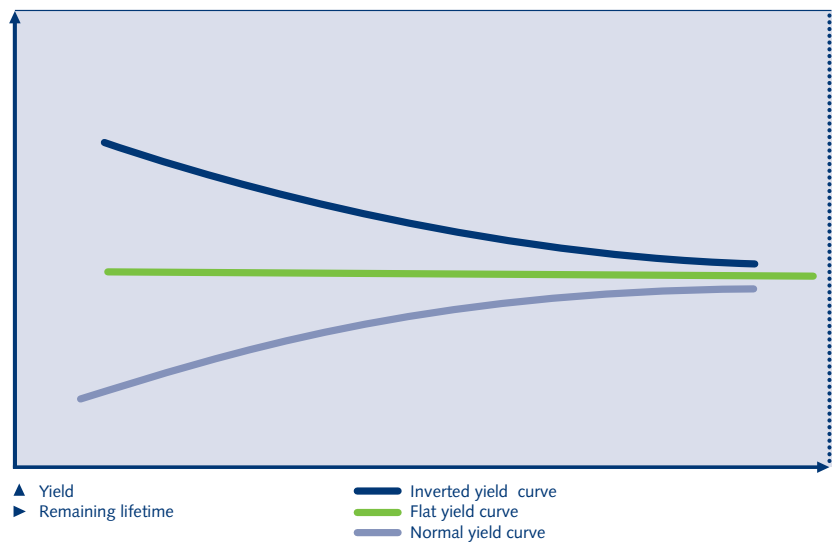
² At this point, we have not yet covered exactly how yields are calculated: for this purpose, we need to take a closer look at the concepts of present value and accrued interest, which we will cover in the following sections.

³ Based on actual/actual.

The Yield Curve

Bond yields are largely dependent on the issuer's creditworthiness and the remaining lifetime of the issue. Since the underlying instruments of Eurex fixed income derivatives are government issues with top-quality credit ratings, the explanations below focus on the correlation between yield and remaining lifetime. These are often presented as a mathematical function – the so-called yield curve. Due to their long-term capital commitment, bonds with a longer remaining lifetime generally tend to yield more than those with a shorter remaining lifetime. This is called a “normal” yield curve. A “flat” yield curve is where all remaining lifetimes have the same rate of interest. An “inverted” yield curve is characterized by a downwards-sloping curve.

Yield Curves



Bond Valuation

In the previous sections, we saw that bonds carry a certain yield for a certain remaining lifetime. These yields may be calculated using the bond's market value (price), coupon payments and redemption (cash flows).

At which market value (price) does the bond yield (actual rate of interest) correspond to prevailing market yields? In the following examples, for clarification purposes, a uniform money market rate (Euribor) is used to represent the market interest rate, although this does not truly reflect circumstances on the capital market.

A bond with annual coupon payments maturing in exactly one year's time is used for this step-by-step explanation. The coupon and the nominal value are repaid at maturity.

Example:

Money market interest rate p.a.	3.63%
Bond	4.5% Federal Republic of Germany debt security due on July 10, 2003
Nominal value	100
Coupon	$4.5\% \times 100 = 4.50$
Valuation date	July 11, 2002 ("today")

This results in the following equation⁴:

$$\text{Present value} = \frac{\text{Nominal value (n)} + \text{Coupon (c)}}{1 + \text{Money market rate (r)}} = \frac{100 + 4.50}{1 + 0.0363} = 100.84$$

To determine the present value of a bond, the future payments are divided by the yield factor ($1 + \text{Money market interest rate}$). This calculation is referred to as "discounting the cash flow". The resulting price is called the present value, since it is generated at the current point in time ("today").

The following example shows the future payments for a bond with a remaining lifetime of three years.

⁴ Cf. Appendix 1 for general formulae.

Example:

Money market interest rate p.a.	3.63%
Bond	4.5% Federal Republic of Germany debt security due on July 11, 2005
Nominal value	100
Coupon	$4.5\% \times 100 = 4.50$
Valuation date	July 12, 2002 ("today")

The bond price can be calculated using the following equation:

$$\text{Present value} = \frac{\text{Coupon (c1)}}{\text{Yield factor}} + \frac{\text{Coupon (c2)}}{(\text{Yield factor})^2} + \frac{\text{Nominal value (n) + coupon (c3)}}{(\text{Yield factor})^3}$$

$$\text{Present value} = \frac{4.50}{(1 + 0.0363)} + \frac{4.50}{(1 + 0.0363)^2} + \frac{100 + 4.50}{(1 + 0.0363)^3} = 102.43$$

When calculating a bond for a date that does not coincide with the coupon payment date, the first coupon needs to be discounted only for the remaining lifetime up until the next coupon payment date. The exponentiation of the yield factor until the bond matures changes accordingly.

Example:

Money market interest rate p.a.	3.63%
Bond	4.5% Federal Republic of Germany debt security due on July 4, 2011
Nominal value	100
Coupon	$4.5\% \times 100 = 4.50$
Valuation date	March 11, 2002 ("today")
Remaining lifetime for the first coupon	115 days or $115/365 = 0.315$ years
Accrued interest	$4.5\% \times 250/365 = 3.08\%$

The annualized interest rate is calculated, on a pro-rata basis, for terms of less than one year. The discount factor is:

$$\frac{1}{1 + (0.0363 \times 0.315)}$$

The interest rate needs to be raised to a higher power for remaining lifetimes beyond one year (1.315, 2.315, 9.315 years). This is also referred to as "compounding" the interest. Accordingly, the bond price is:

$$\text{Present value} = \frac{4.50}{1 + (0.0363 \times 0.315)} + \frac{4.50}{(1 + 0.0363)^{1.315}} + \dots + \frac{4.50 + 100}{(1 + 0.0363)^{9.315}} = 109.84$$

The discount factor for less than one year is also raised to a higher power for the purpose of simplification⁵.

The previous equation can be interpreted in such a way that the present value of the bond equals the sum of its individual present values. In other words, it equals the aggregate of all coupon payments and the repayment of the nominal value. This model can only be used over more than one time period if a constant market interest rate is assumed. The implied flat yield curve tends not to reflect reality. Despite this simplification, determining the present value with a flat yield curve forms the basis for a number of risk indicators. These are described in the following chapters.

One must differentiate between the present value ("dirty price") and the "clean price" when quoting bond prices. According to prevailing convention, the traded price is the "clean price". The "clean price" can be determined by subtracting the accrued interest from the "dirty price". It is calculated as follows:

$$\begin{aligned}\text{Clean price} &= \text{Present value} - \text{Accrued interest} \\ \text{Clean price} &= 109.84 - 3.08 = 106.76\end{aligned}$$

The following section differentiates between a bond's present value and a bond's traded price ("clean price").

A change in market interest rates has a direct impact on the discount factors and hence on the present value of bonds. Based on the example above, this results in the following present value if interest rates increase by one percentage point from 3.63 percent to 4.63 percent:

$$\text{Present value} = \frac{4.50}{(1 + 0.0463)^{0.315}} + \frac{4.50}{(1 + 0.0463)^{1.315}} + \dots + \frac{4.50 + 100}{(1 + 0.0463)^{9.315}} = 102.09$$

The clean price changes as follows:

$$\text{Clean price} = 102.09 - 3.08 = 99.01$$

An increase in interest rates led to a fall of 7.06 percent in the bond's present value from 109.84 to 102.09. The clean price, however, fell by 7.26 percent, from 106.76 to 99.01. The following rule applies to describe the relationship between the present value or the clean price of a bond and interest rate developments:

Bond prices and market yields react inversely to one another.

⁵ Cf. Appendix 1 for general formulae.

Macaulay Duration

In the previous section, we saw how a bond's price was affected by a change in interest rates. The interest rate sensitivity of bonds can also be measured using the concepts of Macaulay duration and modified duration.

The Macaulay duration indicator was developed to analyze the interest rate sensitivity of bonds, or bond portfolios, for the purpose of hedging against unfavorable interest rate changes.

As was previously explained, the relationship between market interest rates and the present value of bonds is inverted: the immediate impact of rising yields is a price loss. Yet, higher interest rates also mean that coupon payments received can be reinvested at more profitable rates, thus increasing the future value of the portfolio. Macaulay duration, which is usually expressed in years, reflects the point in time at which both factors are in balance. It can thus be used to ensure that the sensitivity of a portfolio is in line with a set investment horizon. (Note that the concept is based on the assumption of a flat yield curve, and a "parallel shift" in the yield curve where the yields of all maturities change in the same way.)

Macaulay duration is used to summarize interest rate sensitivity in a single number: changes in the duration of a bond, or duration differentials between different bonds help to gauge relative risks.

The following fundamental relationships describe the characteristics of Macaulay duration: Macaulay duration is lower,

- the shorter the remaining lifetime;
- the higher the market interest rate; and
- the higher the coupon.

Note that a higher coupon actually reduces the riskiness of a bond, compared to a bond with a lower coupon: this is indicated by lower Macaulay duration.

The Macaulay duration of the bond in the previous example is calculated as follows:

Example

Valuation date	March 11, 2002
Security	4.5% Federal Republic of Germany debt security due on July 4, 2011
Money market rate	3.63%
Bond price	109.84

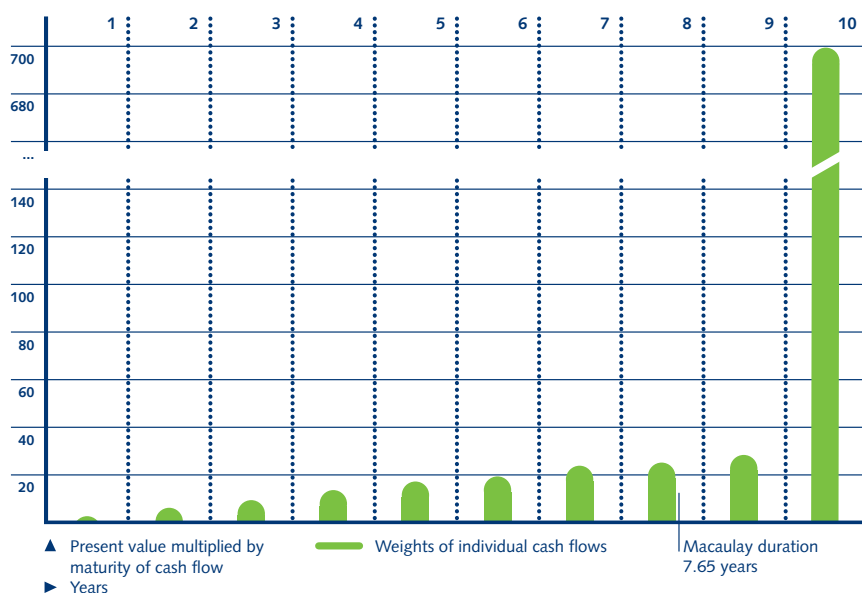
Calculation:

$$\text{Macaulay duration} = \frac{\frac{4.50}{(1 + 0.0363)^{0.315}} \times 0.315 + \frac{4.50}{(1 + 0.0363)^{1.315}} \times 1.315 + \dots + \frac{4.50 + 100}{(1 + 0.0363)^{9.315}} \times 9.315}{109.84}$$

$$\text{Macaulay duration} = 840.51 / 109.84 = 7.65 \text{ years}$$

The 0.315, 1.315, ..., 9.315 factors apply to the remaining lifetimes of the coupons and the repayment of the nominal value. The remaining lifetimes are multiplied by the present value of the individual repayments. Macaulay duration is the aggregate of remaining term of each cash flow, weighted with the share of this cash flow's present value in the overall present value of the bond. Therefore, the Macaulay duration of a bond is dominated by the remaining lifetime of those payments with the highest present value.

Macaulay Duration (Average Remaining Lifetime Weighted by Present Value)



Macaulay duration can also be applied to bond portfolios by accumulating the duration values of individual bonds, weighted according to their share of the portfolio's present value.

Modified Duration

The modified duration is built on the concept of the Macaulay duration. The modified duration reflects the percentage change in the present value (clean price plus accrued interest) given a one unit (one percentage point) change in the market interest rate. The modified duration is equivalent to the negative value of the Macaulay duration, discounted over a period of time:

$$\text{Modified duration} = - \frac{\text{Duration}}{1 + \text{Yield}}$$

The modified duration for the example above is:

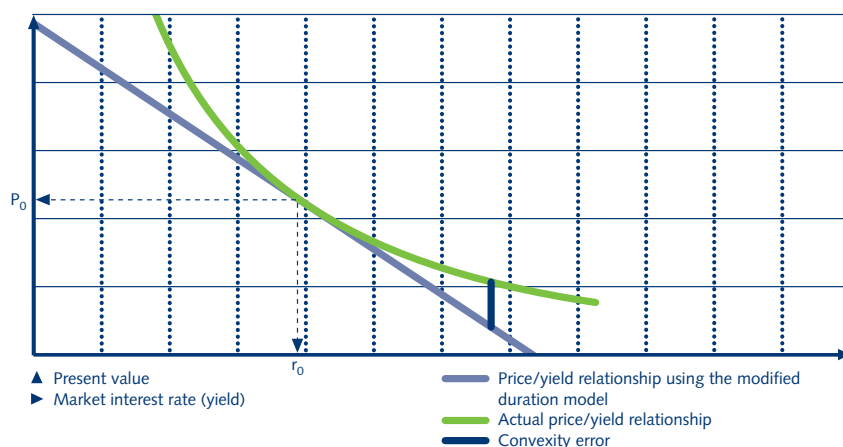
$$\text{Modified duration} = - \frac{7.65}{1 + 0.0363} = -7.38\%$$

According to the modified duration model, a one percentage point rise in the interest rate should lead to a 7.38 percent fall in the present value.

Convexity – the Tracking Error of Duration

Despite the validity of the assumptions referred to in the previous section, calculating the change in value by means of the modified duration tends to be imprecise due to the assumption of a linear correlation between the present value and interest rates. In general, however, the price/yield relationship of bonds tends to be convex, and therefore, a price increase calculated by means of the modified duration is under- or overestimated, respectively.

Relationship between Bond Prices and Capital Market Interest Rates



In general, the greater the changes in the interest rate, the more imprecise the estimates for present value changes will be using modified duration. In the example used, the new calculation resulted in a fall of 7.06 percent in the bond's present value, whereas the estimate using the modified duration was 7.38 percent. The inaccuracies resulting from non-linearity when using the modified duration can be corrected by means of the so-called convexity formula.

Compared to the modified duration formula, each element of the summation in the numerator is multiplied by $(1 + t_{c1})$ and the given denominator by $(1 + t_{r_{c1}})^2$ when calculating the convexity factor.

The calculation below uses the same previous example:

$$\text{Convexity} = \frac{\frac{4.50}{(1 + 0.0363)^{0.315}} \times 0.315 \times (0.315 + 1) + \frac{4.50}{(1 + 0.0363)^{1.315}} \times 1.315 \times (1.315 + 1) + \dots + \frac{100 + 4.50}{(1 + 0.0363)^{9.315}} \times 9.315 \times (9.315 + 1)}{(1 + 0.0363)^2 \times 109.84} = 69.15$$

This convexity factor is used in the following equation:

$$\text{Percentage present value change of bond} = \text{Modified duration} \times \text{Change in market rates} + 0.5 \times \text{Convexity} \times (\text{Change in market rates})^2$$

An increase in the interest rate from 3.63 percent to 4.63 percent would result in:

$$\text{Percentage present value change of bond} = -7.38 \times (0.01) + 0.5 \times 69.15 \times (0.01)^2 = -0.0703 = -7.03\%$$

The results of the three calculation methods are compared below:

Calculation method:	Results
Recalculating the present value	-7.06%
Projection using modified duration	-7.38%
Projection using modified duration and convexity	-7.03%

This illustrates that taking the convexity into account provides a result similar to the price arrived at in the recalculation, whereas the estimate using the modified duration deviates significantly. However, one should note that a uniform interest rate was used for all remaining lifetimes (flat yield curve) in all three examples.

Eurex Fixed Income Derivatives

Characteristics of Exchange-traded Financial Derivatives

Introduction

Contracts for which the prices are derived from underlying cash market securities or commodities (which are referred to as “underlying instruments” or “underlyings”) such as equities, bonds or oil, are known as derivative instruments or simply derivatives. Trading derivatives is distinguished by the fact that settlement takes place on specific dates (settlement date). Whereas payment against delivery for cash market transactions must take place after two or three days (settlement period), exchange-traded futures and options contracts, with the exception of exercising options, may provide for settlement on just four specific dates during the year.

Derivatives are traded both on organized derivatives exchanges such as Eurex and in the over-the-counter (OTC) market. For the most part, standardized contract specifications and the process of marking to market or margining via a clearing house distinguish exchange-traded products from OTC derivatives. Eurex lists futures and options on financial instruments.

Flexibility

Organized derivatives exchanges provide investors with the facilities to enter into a position based on their market perception and in accordance with their appetite for risk, but without having to buy or sell any securities. By entering into a counter transaction they can neutralize (“close out”) their position prior to the contract maturity date. Any profits or losses incurred on open positions in futures or options on futures are credited or debited on a daily basis.

Transparency and Liquidity

Trading standardized contracts results in a concentration of order flows thus ensuring market liquidity. Liquidity means that large amounts of a product can be bought and sold at any time without excessive impact on prices. Electronic trading on Eurex guarantees extensive transparency of prices, volumes and executed transactions.

Leverage Effect

When entering into an options or futures trade, it is not necessary to pay the full value of the underlying instrument up front. Hence, in terms of the capital invested or pledged, the percentage profit or loss potential for these forward transactions is much greater than for the actual bonds or equities.

Introduction to Fixed Income Futures

What are Fixed Income Futures? – Definition

Fixed income futures are standardized forward transactions between two parties, based on fixed income instruments such as bonds with coupons. They comprise the obligation –

... to purchase	Buyer	Long future	Long future
... or to deliver	Seller	Short future	Short future
... a given financial instrument	Underlying instrument	German Government Bonds	Swiss Confederation Bonds
... with a given remaining lifetime		8.5-10.5 years	8-13 years
... in a set amount	Contract size	EUR 100,000 nominal	CHF 100,000 nominal
... at a set point in time	Maturity	March 10, 2002	March 10, 2002
... at a determined price	Futures price	106.00	120.50

Eurex fixed income derivatives are based upon the delivery of an underlying bond which has a remaining maturity in accordance with a predefined range. The contract's deliverable list will contain bonds with a range of different coupon levels, prices and maturity dates. To help standardize the delivery process the concept of a notional bond is used. See the section below on contract specification and conversion factors for more detail.

Futures Positions – Obligations

A futures position can either be “long” or “short”:

Long position Buying a futures contract The buyer's obligations: At maturity, a long position automatically results in the obligation to buy deliverable bonds: The obligation to buy the interest rate instrument relevant to the contract at the price determined on the delivery date.	Short position Selling a futures contract The seller's obligations: At maturity, a short position automatically results in the obligation to deliver such bonds: The obligation to deliver the interest rate instrument relevant to the contract at the price determined on the delivery date.
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Settlement or Closeout

Futures are generally settled by means of a cash settlement or by physically delivering the underlying instrument. Eurex fixed income futures provide for the physical delivery of securities. The holder of a short position is obliged to deliver either long-term Swiss Confederation Bonds or short-, medium- or long-term German Government debt securities, depending on the traded contract. The holder of the corresponding long position must accept delivery against payment of the delivery price.

Securities of the respective issuers whose remaining lifetime on the futures delivery date is within the parameters set for each contract, can be delivered. These parameters are also known as the maturity ranges for delivery. The choice of bond to be delivered must be notified (the notification obligation of the holder of the short position). The choice and valuation of a bond at settlement is described in the section on “Bond Valuation”.

However, it is worth noting that when entering into a futures position it is not necessarily based upon the intention to actually deliver, or take delivery of, the underlying instruments at maturity. For instance, futures are designed to track the price development of the underlying instrument during the lifetime of the contract. In the event of a price increase in the futures contract, an original buyer of a futures contract is able to realize a profit by simply selling an equal number of contracts to those originally bought. The reverse applies to a short position, which can be closed out by buying back futures.

As a result, a noticeable reduction in the open interest (the number of open long and short positions in each contract) occurs in the days prior to maturity of a bond futures contract. Whilst during the contract's lifetime, open interest may well exceed the volume of deliverable bonds available, this figure tends to fall considerably as soon as open interest starts shifting from the shortest delivery month to the next, prior to maturity (a process known as “rollover”).

Contract Specifications

Information on the detailed contract specifications of fixed income futures traded at Eurex can be found in the "Eurex Products" brochure or on the Eurex homepage www.eurexchange.com. The most important specifications of Eurex fixed income futures are detailed in the following example based on Euro Bund Futures and CONF Futures.

An investor buys:

... 2	Contracts	The futures transaction is based on a nominal value of 2 x EUR 100,000 of deliverable bonds for the Euro Bund Future, or 2 x CHF 100,000 of deliverable bonds for the CONF Future.
... June 2002	Maturity month	The next three quarterly months within the cycle March/June/September/December are available for trading. Thus, the Euro Bund and CONF Futures have a maximum remaining lifetime of nine months. The last trading day is two exchange trading days before the 10th calendar day (delivery day) of the maturity month.
Euro Bund or CONF Futures, respectively	Underlying instrument	The underlying instrument for Euro Bund Futures is a 6% notional long-term German Government Bond. For CONF Futures it is a Swiss Confederation Bond.
... at 106.00 or 120.50, respectively	Futures price	The futures price is quoted in percent, to two decimal points, of the nominal value of the underlying bond. The minimum price change (tick) is EUR 10.00 or CHF 10.00 (0.01%).

In this example, the buyer is obliged to buy either German Government Bonds or Swiss Confederation Bonds, which are included in the basket of deliverable bonds, to a nominal value of EUR or CHF 200,000, in June 2002.

Eurex Fixed Income Futures – Overview

The specifications of fixed income futures are largely distinguished by the baskets of deliverable bonds that cover different maturity ranges. The corresponding remaining lifetimes are set out in the following table:

Underlying instrument: German Government debt securities	Nominal contract value	Remaining lifetime of the deliverable bonds	Product code
Euro Schatz Future	EUR 100,000	1 ³ / ₄ to 2 ¹ / ₄ years	FGBS
Euro Bobl Future	EUR 100,000	4 ¹ / ₂ to 5 ¹ / ₂ years	FGBM
Euro Bund Future	EUR 100,000	8 ¹ / ₂ to 10 ¹ / ₂ years	FGBL
Euro Buxl Future	EUR 100,000	20 to 30 ¹ / ₂ years	FGBX

Underlying instrument: Swiss Confederation Bonds	Nominal contract value	Remaining lifetime of the deliverable bonds	Product code
CONF Future	CHF 100,000	8 to 13 years	CONF

Futures Spread Margin and Additional Margin

When a futures position is created, cash or other collateral is deposited with Eurex Clearing AG – the Eurex clearing house. Eurex Clearing AG seeks to provide a guarantee to all clearing members in the event of a member defaulting. This additional margin deposit is designed to protect the clearing house against a forward adverse price movement in the futures contract. The clearing house is the ultimate counterparty in all Eurex transactions and must safeguard the integrity of the market in the event of a clearing member default.

Offsetting long and short positions in different maturity months of the same futures contract are referred to as spread positions. The high correlation of these positions means that the spread margin rates are lower than those for additional margin. Additional margin is charged for all non-spread positions. Margin collateral must be pledged in the form of cash or securities.

A detailed description of margin requirements calculated by the Eurex clearing house (Eurex Clearing AG) can be found in the brochure on “Risk Based Margining”.

Variation Margin

A common misconception regarding bond futures is that when delivery of the actual bonds are made, they are settled at the original opening futures price. In fact delivery of the actual bonds is made using a final futures settlement price (see the section below on conversion factor and delivery price). The reason for this is that during the life of a futures position, its value is marked to market each day by the clearing house in the form of variation margin. Variation margin can be viewed as the futures contract's profit or loss, which is paid and received each day during the life of an open position. The following examples illustrate the calculation of the variation margin, whereby profits are indicated by a positive sign, losses by a negative sign.

Calculating the variation margin for a new futures position:

Daily futures settlement price
– Futures purchase or selling price
= Variation margin

The daily settlement price of the CONF Future in our example is 121.65. The contracts were bought at a price of 121.50.

Example – CONF variation margin:

CHF 121,650 (121.65% of CHF 100,000)
– CHF 121,500 (121.50% of CHF 100,000)
= CHF 150

On the first day, the buyer of the CONF Future makes a profit of CHF 150 per contract (0.15 percent of the nominal value of CHF 100,000), that is credited via the variation margin. Alternatively the calculation can be described as the difference between $121.65 - 121.50 = 15$ ticks. The futures contract is based upon CHF 100,000 nominal of bonds, so the value of a small price movement (tick) of CHF 0.01 equates to CHF 10 (i.e. $1,000 \times 0.01$). This is known as the tick value. Therefore the profit on the one futures trade is $15 \times \text{CHF } 10 \times 1 = \text{CHF } 150$.

The same process applies to the Euro Bund Future. The Euro Bund Futures daily settlement price is 105.70. It was bought at 106.00.

The variation margin calculation results in the following:

Example – Euro Bund variation margin:	
EUR	105,700 (105.70% of EUR 100,000)
EUR	–106,000 (106.00% of EUR 100,000)
= EUR	–300

The buyer of the Euro Bund Futures incurs a loss of EUR 300 per contract (0.3 percent of the nominal value of EUR 100,000), that is consequently debited by way of variation margin. Alternatively $105.70 - 106.00 = 30$ ticks loss multiplied by the tick value of one bund future (EUR 10) = EUR –300.

Calculating the variation margin during the contract's lifetime:	
	Futures daily settlement price on the current exchange trading day
–	Futures daily settlement price on the previous exchange trading day
=	Variation margin

Calculating the variation margin when the contract is closed out:	
	Futures price of the closing transaction
–	Futures daily settlement price on the previous exchange trading day
=	Variation margin

The Futures Price – Fair Value

While the chapter "Bond Valuation" focused on the effect of changes in interest rate levels on the present value of a bond, this section illustrates the relationship between the futures price and the value of the corresponding deliverable bonds.

An investor who wishes to acquire bonds on a forward date can either buy a futures contract today on margin, or buy the cash bond and hold the position over time. Buying the cash bond involves an actual financial cost which is offset by the receipt of coupon income (accrued interest). The futures position on the other hand, over time, has neither the financing costs nor the receipts of an actual long spot bond position (cash market).

Therefore to maintain market equilibrium, the futures price must be determined in such a way that both the cash and futures purchase yield identical results. Theoretically, it should thus be impossible to realize risk-free profits using counter transactions on the cash and forward markets (arbitrage).

Both investment strategies are compared in the following table:

Time	Period	Futures purchase investment/valuation	Cash bond purchase investment/valuation
Today		Entering into a futures position (no cash outflow)	Bond purchase (market price plus accrued interest)
	Futures lifetime	Investing the equivalent value of the financing cost saved, on the money market	Coupon credit (if any) and money market investment of the equivalent value
Futures delivery		Portfolio value Bond (purchased at the futures price) + income from the money market investment of the financing costs saved	Portfolio value Value of the bond including accrued interest + any coupon credits + any interest on the coupon income

Taking the factors referred to above into account, the futures price is derived in line with the following general relationship⁶:

$$\text{Futures price} = \text{Cash price} + \text{Financing costs} - \text{Proceeds from the cash position}$$

Which can be expressed mathematically as⁷:

$$\text{Futures price} = C_t + (C_t + c \frac{t-t_0}{365}) \times {}_t r_c \times \frac{T-t}{360} - c \times \frac{T-t}{365}$$

Whereby:

C_t : Current clean price of the underlying security (at time t)

c : Bond coupon (percent; actual/actual for euro-denominated bonds)

t_0 : Coupon date

t : Value date

${}_t r_c$: Short-term funding rate (percent; actual/360)

T : Futures delivery date

$T-t$: Futures remaining lifetime (days)

⁶ Readers should note that the formula shown here has been simplified for the sake of transparency; specifically, it does not take into account the conversion factor, interest on the coupon income, borrowing cost/lending income or any diverging value date conventions in the professional cash market.

⁷ Please note that the number of days in the year (denominator) depends on the convention in the respective markets. Financing costs are usually calculated based on the money market convention (actual/360), whereas the accrued interest and proceeds from the cash positions are calculated on an actual/actual basis, which is the market convention for all euro-denominated government bonds.

Cost of Carry and Basis

The difference between the financing costs and the proceeds from the cash position – coupon income is referred to as the “cost of carry”. The futures price can also be expressed as follows⁸:

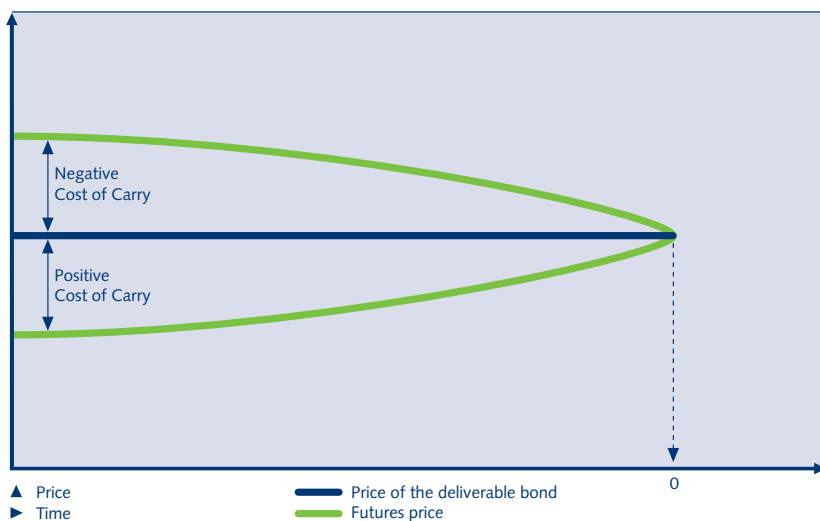
$$\text{Price of the deliverable bond} = \text{Futures price} + \text{Cost of carry}$$

The basis is the difference between the bond price in the cash market (expressed by the prices of deliverable bonds) and the futures price, and is thus equivalent to the following:

$$\text{Price of the deliverable bond} = \text{Futures price} + \text{Basis}$$

The futures price is either higher or lower than the price of the underlying instrument, depending on whether the cost of carry is negative or positive. The basis diminishes with approaching maturity. This effect is called “basis convergence” and can be explained by the fact that as the remaining lifetime decreases, so do the financing costs and the proceeds from the bonds. The basis equals zero at maturity. The futures price is then equivalent to the price of the underlying instrument – this effect is called “basis convergence”.

Basis Convergence (Schematic)



The following relationships apply:

Financing costs > Proceeds from the cash position: → **Negative** cost of carry
Financing costs < Proceeds from the cash position: → **Positive** cost of carry

⁸ Cost of carry and basis are frequently shown in literature using a reverse sign.

Conversion Factor (Price Factor) and Cheapest-to-Deliver (CTD) Bond

The bonds eligible for delivery are non-homogeneous – although they have the same issuer, they vary by coupon level, maturity and therefore price.

At delivery the conversion factor is used to help calculate a final delivery price. Essentially the conversion factor generates a price at which a bond would trade if its yield were 6% on delivery day. One of the assumptions made in the conversion factor formula is that the yield curve is flat at the time of delivery, and what is more, it is at the same level as that of the futures contract's notional coupon. Based on this assumption the bonds in the basket for delivery should be virtually all equally deliverable. (Of course, this does not truly reflect reality: we will discuss the consequences below.)

The delivery price of the bond is calculated as follows:

Delivery price = Final settlement price of the future × Conversion factor of the bond + Accrued interest of the bond

Calculating the number of interest days for issues denominated in Swiss francs and euros is different (Swiss francs: 30/360; euros: actual/actual), resulting in two diverging conversion factor formulae. These are included in the appendices. The conversion factor values for all deliverable bonds are displayed on the Eurex website www.eurexchange.com.

The conversion factor (CF) of the bond delivered is incorporated as follows in the futures price formula (see p. 25 for an explanation of the variables used):

$$\text{Theoretical futures price} = \frac{1}{CF} \left[C_t + \left(C_t + c \frac{t - t_0}{365} \right) \times {}_t r_c \times \frac{T - t}{360} - c \times \frac{T - t}{365} \right]$$

The following example describes how the theoretical price of the Euro Bund Future June 2002 is calculated.

Example:

Trade date	May 3, 2002
Value date	May 8, 2002
Cheapest-to-deliver bond	3.75% Federal Republic of Germany debt security due on January 4, 2011
Price of the cheapest-to-deliver	90.50
Futures delivery date	June 10, 2002
Accrued interest	$3.75\% \times (124/365) \times 100 = 1.27$
Conversion factor of the CTD	0.852420
Money market rate	3.63%

$$\text{Theoretical futures price} = \frac{1}{0.852420} \left[90.50 + (90.50 + 1.27) \times 0.0363 \times \frac{33}{360} - 3.75 \times \frac{33}{365} \right]$$

$$\text{Theoretical futures price} = \frac{1}{0.852420} \left[90.50 + 0.3054 - 0.3390 \right]$$

$$\text{Theoretical futures price} = 106.13$$

In reality the actual yield curve is seldom the same as the notional coupon level; also, it is not flat as implied by the conversion factor formula. As a result, the implied discounting at the notional coupon level generally does not reflect the true yield curve structure.

The conversion factor thus inadvertently creates a bias which promotes certain bonds for delivery above all others. The futures price will track the price of the deliverable bond that presents the short futures position with the greatest advantage upon maturity. This bond is called the cheapest to deliver (or "CTD"). In case the delivery price of a bond is higher than its market valuation, holders of a short position can make a profit on the delivery, by buying the bond at the market price and selling it at the higher delivery price. They will usually choose the bond with the highest price advantage. Should a delivery involve any price disadvantage, they will attempt to minimize this loss.

Identifying the Cheapest-to-Deliver Bond

On the delivery day of a futures contract, a trader should not really be able to buy bonds in the cash bond market, and then deliver them immediately into the futures contract at a profit – if he/she could do this it would result in a cash and carry arbitrage. We can illustrate this principle by using the following formula and examples.

$$\text{Basis} = \text{Cash bond price} - (\text{Futures price} \times \text{Conversion factor})$$

At delivery, basis will be zero. Therefore, at this point we can manipulate the formula to achieve the following relationship:

$$\frac{\text{Cash bond price}}{\text{Conversion factor}} = \text{Futures price}$$

This futures price is known as the zero basis futures price. The following table shows an example of some deliverable bonds (note that we have used hypothetical bonds for the purposes of illustrating this effect). At a yield of 5% the table records the cash market price at delivery and the zero basis futures price (i.e. cash bond price divided by the conversion factor) of each bond.

Zero basis futures price at 5% yield

Coupon	Maturity	Conversion factor	Price at 5% yield	Price divided by conversion factor
5%	07/15/2012	0.925836	99.99	108.00
6%	03/04/2012	0.999613	107.54	107.58
7%	05/13/2011	1.067382	114.12	106.92

We can see from the table that each bond has a different zero basis futures price, with the 7% 05/13/2011 bond having the lowest zero basis futures price of 106.92. In reality of course only one real futures price exists at delivery. Suppose that at delivery the real futures price was 106.94. If that was the case an arbitrageur could buy the cash bond (7% 05/13/02) at 114.12 and sell it immediately via the futures market at 106.94×1.067382 and receive 114.14. This would create an arbitrage profit of 2 ticks. Neither of the two other bonds would provide an arbitrage profit, however, with the futures at 106.94. Accrued interest is ignored in this example as the bond is bought and sold into the futures contract on the same day.

It follows that the bond most likely to be used for delivery is always the bond with the lowest zero basis futures price – the cheapest cash bond to purchase in the cash market in order to fulfil a short delivery into the futures contract, i.e. the CTD bond.

Extending the example further, we can see how the zero basis futures prices change under different market yields and how the CTD is determined.

Zero basis futures price at 5%, 6%, 7% yield

Coupon	Maturity	Conversion factor	Price at 5%	Price / CF	Price at 6%	Price / CF	Price at 7%	Price / CF
5%	07/15/2012	0.925836	99.99	107.99	92.58	100.00	85.84	92.72
6%	03/04/2012	0.999613	107.54	107.58	99.97	100.00	93.07	93.11
7%	05/13/2011	1.067382	114.12	106.92	106.75	100.01	99.99	93.68

- If the market yield is above the notional coupon level, bonds with a **longer duration** (lower coupon given similar maturities / longer maturity given similar coupons) will be preferred for delivery.
- If the market yield is below the notional coupon level, bonds with a **shorter duration** (higher coupon given similar maturities / shorter maturity given similar coupons) will be preferred for delivery.
- When yields are at the notional coupon level (6%) the bonds are almost all equally preferred for delivery.

As we pointed out above, this bias is caused by the “incorrect” discount rate of 6 percent implied by the way the conversion factor is calculated. For example, when market yields are below the level of the notional coupon, all eligible bonds are undervalued in the calculation of the delivery price. This effect is least pronounced for bonds with a low duration as these are less sensitive to variations of the discount rate (market yield)⁹. So, if market yields are below the implied discount rate (i.e. the notional coupon rate), low duration bonds tend to be cheapest-to-deliver. This effect is reversed for market yields above 6 percent.

⁹ Cf. chapters “Macaulay Duration” and “Modified Duration”.

The graph below shows a plot of the three deliverable bonds, illustrating how the CTD changes as the yield curve shifts.

Identifying the CTD under different market conditions



Applications of Fixed Income Futures

There are three motives for using derivatives: trading, hedging and arbitrage.

Trading involves entering into positions on the derivatives market for the purpose of making a profit, assuming that market developments are forecast correctly. Hedging means securing the price of an existing or planned portfolio. Arbitrage is exploiting price imbalances to achieve risk-free profits.

To maintain the balance in the derivatives markets it is important that both traders and hedgers are active thus providing liquidity. Trades between hedgers can also take place, whereby one counterparty wants to hedge the price of an existing portfolio against price losses and the other the purchase price of a future portfolio against expected price increases. The central role of the derivatives markets is the transfer of risk between these market participants. Arbitrage ensures that the market prices of derivative contracts diverge only marginally and for a short period of time from their theoretically correct values.

Trading Strategies

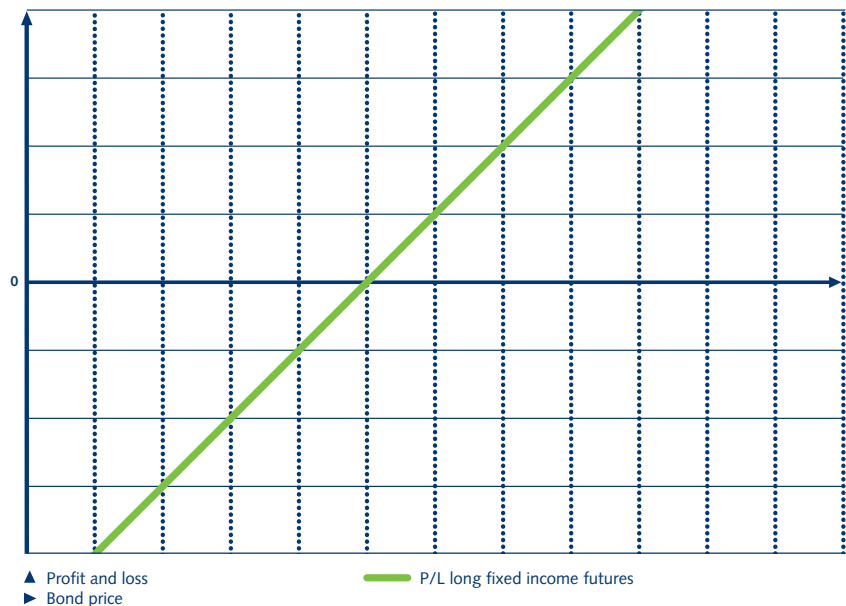
Basic Futures Strategies

Building exposure by using fixed income futures has the attraction of allowing investors to benefit from expected interest rate moves without having to tie up capital by buying bonds. For a futures position, contrary to investing on the cash market, only additional margin needs to be pledged (cf. chapter “Futures Spread Margin and Additional Margin”). Investors incurring losses on their futures positions – possibly as a result of incorrect market forecasts – are obliged to settle these losses immediately, and in full (variation margin). This could amount to a multiple of the amount pledged. The change in value relative to the capital “invested” is consequently much higher than for a similar cash market transaction. This is called the “leverage effect”. In other words, the substantial profit potential associated with a straight fixed income future position is reflected by the significant risks involved.

Long Positions ("Bullish" Strategies)

Investors expecting falling market yields for a certain remaining lifetime will decide to buy futures contracts covering this section of the yield curve. If the forecast turns out to be correct, a profit is made on the futures position. As is characteristic for futures contracts, the profit potential on such a long position is proportional to its risk exposure. In principle, the price/yield relationship of a fixed income futures contract corresponds to that of a portfolio of deliverable bonds.

Profit and Loss Profile on the Last Trading Day, Long Fixed Income Futures



Rationale

The investor wants to benefit from a forecast development without tying up capital in the cash market.

Initial Situation

The investor assumes that yields on German Federal Debt Obligations (Bundesobligationen) will fall.

Strategy

The investor buys 10 Euro Bobl Futures June 2002 at a price of 105.10, with the intention to close out the position during the contract's lifetime. If the price of the Euro Bobl Futures rises, the investor makes a profit on the difference between the purchase price and the higher selling price. Constant analysis of the market is necessary to correctly time the position exit by selling the contracts.

The calculation of additional and variation margins for a hypothetical price development

is illustrated in the following table. The additional margin is derived by multiplying the margin parameter, as set by Eurex Clearing AG (in this case EUR 1,000 per contract), by the number of contracts.

Date	Transaction	Purchase/ selling price	Daily settlement price	Variation margin ¹⁰ profit in EUR	Variation margin loss in EUR	Additional margin ¹¹ in EUR
03/11	Buy 10 Euro Bobl Futures June 2002	105.10	104.91		1,900	-10,000
03/12			104.97	600		
03/13			104.80		1,700	
03/14			104.69		1,100	
03/15			104.83	1,400		
03/18			105.14	3,100		
03/19			105.02		1,200	
03/20	Sell 10 Euro Bobl Futures June 2002	105.37		3,500		
03/21						+10,000
Total		0.27		8,600	5,900	0

Changed Market Situation:

The investor closes out the futures position at a price of 105.37 on March 20.
The additional margin pledged is released the following day.

Result:

The proceeds of EUR 2,700 made on the difference between the purchase and sale is equivalent to the balance of the variation margin (EUR 8,600 – EUR 5,900) calculated on a daily basis. Alternatively the net profit is the sum of the futures price movement multiplied by 10 contracts multiplied by the value of EUR 10: $(105.37 - 105.10) \times 10 \times \text{EUR } 10 = \text{EUR } 2,700$.

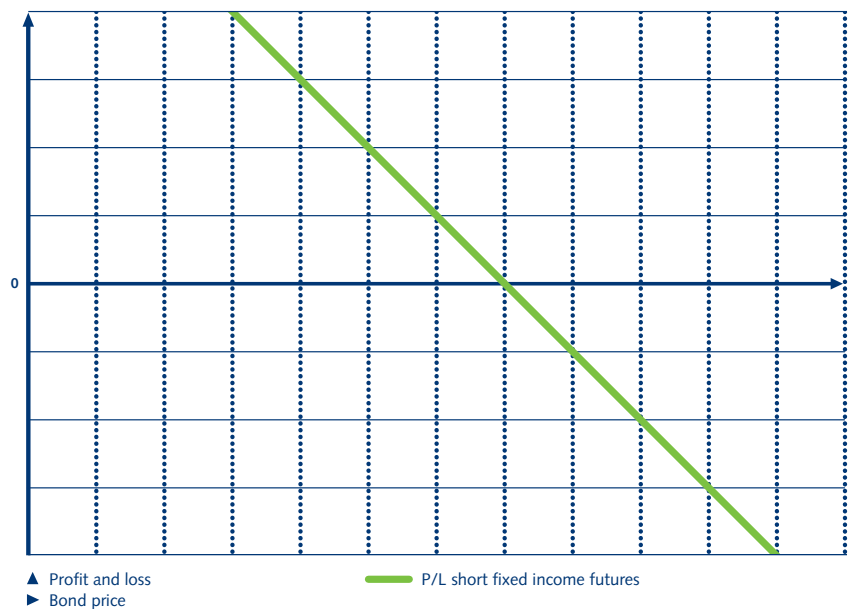
¹⁰ Cf. chapter "Variation Margin".

¹¹ Cf. chapter "Futures Spread Margin and Additional Margin".

Short Positions ("Bearish" Strategies)

Investors who expect market yields to rise sell futures contracts. The short fixed income futures diagram illustrates the outcome of the futures price and its corresponding profit and loss potential.

Profit and Loss Profile on the Last Trading Day, Short Fixed Income Futures



Rationale

The investor wishes to benefit from rising yields, but is unable to sell the actual bonds short (i.e. sell them without owning them).

Initial Situation

The investor expects yields for short-term (two-year) German Federal Treasury Notes (Bundesschatzanweisungen) to rise.

Strategy

The investor decides to enter into a short position of 20 Euro Schatz Futures June 2002 contracts at a price of 102.98. After a certain period of time, he/she closes out this position by buying back the contracts. Again, the additional margin is derived by multiplying the margin parameter, as set by Eurex Clearing AG (in this case EUR 500 per contract), by the number of contracts.

Date	Transaction	Purchase/ selling price	Daily settlement price	Variation margin ¹² profit in EUR	Variation margin loss in EUR	Additional margin ¹³ in EUR
03/11	Sell 20 Euro Schatz Futures June 2002	102.98	103.00		400	-10,000
03/12			102.60	8,000		
03/13			102.48	2,400		
03/14			102.52		800	
03/15			103.20		13,600	
03/18			103.45		5,000	
03/19			103.72		5,400	
03/20	Buy 20 Euro Schatz Futures June 2002	103.60		2,400		
03/21						+10,000
Total		- 0.62		12,800	25,200	0

Changed Market Situation

The investor closes out the futures position at a price of 103.60 on March 20.
The additional margin pledged is released the following day.

Result

The loss of EUR 12,400 is equivalent to the sum of the variation margin cash flows (EUR 12,800 – EUR 25,200) calculated on a daily basis.

Alternatively the net result is the sum of the futures price movement multiplied by 20 contracts multiplied by the value of EUR 10:

$$(102.98 - 103.60) \times 20 \times \text{EUR } 10 = \text{EUR } -12,400$$

Spread Strategies

A spread is the simultaneous purchase and sale of futures contracts. Spread positions are designed to achieve profits on expected changes in the price difference between the long and short positions.

There are various types of spreads. This section describes time spreads and inter-product spreads.

¹² Cf. chapter "Variation Margin".

¹³ Cf. chapter "Futures Spread Margin and Additional Margin".

Time Spread

A time spread comprises long and short positions in futures with the same underlying instrument but with different maturity dates. This strategy is based on one of two assumptions: the first motivation is a forecast change in the price difference between both contracts, because of expected shifts in the financing costs for different maturities. Alternatively, a spread position could be based on the perception of mispricing of one contract (or both contracts) and the expectation that this mispricing will be corrected by the market. The simultaneous establishment of long and short positions is subject to lower risks compared to outright long or short positions. Even when the expectations are not met, the loss incurred on one futures position is largely offset by the counter position. This is why Eurex Clearing AG demands reduced margin rates for time spread positions (Futures Spread Margin instead of Additional Margin).

Time Spread	
Purchase Simultaneous purchase of a fixed income futures contract with a shorter lifetime and the sale of the same futures contract with a longer lifetime	Sale Simultaneous sale of a fixed income futures contract with a shorter lifetime and the purchase of the same futures contract with a longer lifetime
... where a positive (negative) spread between the shorter and the longer maturity is expected to widen (narrow); or	... where a positive (negative) spread between the shorter and the longer maturity is expected to narrow (widen); or
... where the contract with the longer lifetime is overvalued in relative terms.	... where the contract with the shorter lifetime is overvalued in relative terms.

Rationale

An investor analyzes the value of the September 2002 Euro Bobl Future in April and establishes that the contract is overvalued. The investor expects the spread between the June and September Euro Bobl Futures to widen.

Initial Situation

Valuation date	April 18, 2002 ("today")
Euro Bobl Futures June 2002	104.81
Euro Bobl Futures September 2002	104.75

Strategy

Purchase of 5 Euro Bobl Futures June/September 2002 time spreads.

Euro Bobl Futures June 2002 bought at a price of	– 104.81
Euro Bobl Futures September 2002 sold at a price of	+ 104.75
Price of June/September spread bought	– 0.06

Changed Market Situation

The investor's expectations have come true in May. He/she decides to close out the spread position and to realize his/her profit.

Euro Bobl Futures June 2002 sold at a price of	+ 105.34
Euro Bobl Futures September 2002 bought at a price of	– 104.99
Price of June/September spread sold	+ 0.35

Result

June/September spread entry level	– 0.06
June/September spread closeout level	+ 0.35
Result per contract	+ 0.29

The total profit for 5 contracts is $5 \times 29 \times \text{EUR } 10 = \text{EUR } 1,450.00$.

Inter-Product Spread

Inter-product-spreads involve long and short futures positions with different underlying instruments. This type of strategy is directed at varying yield developments in the respective maturity sectors. Assuming a normal yield curve, if 10-year yields rises more than 5-year or 2-year yields, the yield curve is "steepening", whereas a "flattening" indicates approaching short-, medium- and long-term yields.

In comparison to outright position trading, inter-product spreads are also subject to lower risks. When calculating additional margin, the correlation of the price development is taken into account by the fact that the Euro Bund and the Euro Bobl Futures form part of the same margin group¹⁴.

Due to the different interest rate sensitivities of bonds with different remaining lifetimes and the corresponding futures contracts, the long and short positions (the "legs" of the strategy) must be weighted according to the modified duration of the contracts. Otherwise parallel shifts in the yield curve would lead to a change in the value of the spread.

¹⁴ Cf. Risk Based Margining brochure.

Inter-Product Spread	
Purchase Simultaneous purchase of a fixed income future on a shorter-term underlying instrument and sale of a fixed income future on a longer-term underlying instrument, with identical or similar maturities ... the yield curve is expected to steepen.	Sale Simultaneous sale of a fixed income future on a shorter-term underlying instrument and the purchase of a fixed income future on a longer-term underlying instrument, with identical or similar maturities ... the yield curve is expected to flatten.

Motive

In mid-May, assuming a normal relationship between the five and ten-year sectors, the investor expects the yield curve to become steeper. This means that long-term yields will rise more (or fall less) than medium-term yields.

Initial Situation

Valuation date	May 13, 2002
Euro Bobl Futures June 2002	104.84
Euro Bund Futures June 2002	106.00
Euro Bobl/Euro Bund ratio	5:3

Strategy

By buying in the ratio of 10 Euro Bobl Futures and simultaneously selling 6 Euro Bund Futures, the investor wants to benefit from the forecast interest rate development. Due to the different interest rate sensitivities of the respective issues, the medium-term and long-term positions are weighted differently. The strategy's success depends mainly on the yield differential and not on the absolute level of market yields.

Changed Market Situation

10-year yields have risen by twenty basis points, compared to ten basis points in the 5-year sector by the beginning of June. The Euro Bund and Euro Bobl Futures prices have developed as follows:

Valuation date	June 6, 2002
Euro Bobl Futures June 2002	104.40
Euro Bund Futures June 2002	104.52

The investor decides to close out his/her position, and obtains the following result:

Result from the Euro Bobl position		EUR
Euro Bobl Futures June 2002 bought at a price of	- 104.84	- 104,840
Euro Bobl Futures June 2002 sold at a price of	+ 104.40	+ 104,400
Loss per contract		- 440
Loss incurred on the Euro Bobl position (10 contracts)		- 4,400

Result from the Euro Bund position		EUR
Euro Bund Futures June 2002 sold at a price of	106.00	106,000
Euro Bund Futures June 2002 bought at a price of	- 104.52	- 104,520
Profit per contract		1,480
Profit made on the Euro Bund position (6 contracts)		8,880

Total result in EUR	- 4,400 + 8,880 = 4,480	
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The investor made a total profit of EUR 4,480.

Hedging Strategies

Potential hedgers who hold a long (short) position in the cash bond market will seek to protect their position from short term adverse movements in yields, by using futures. Depending on the position to be hedged, they will buy or sell futures and thus, in effect, fix a future price level for their underlying position.

Hedging of interest rate positions largely comprises choosing a suitable futures contract, determining the number of contracts required to hedge the cash position (the "hedge ratio") and deciding on the potential adjustments to be made to the hedge ratio during the period in question.

Choice of the Futures Contract

Ideally, a futures contract is used to hedge securities that belong to the basket of deliverable bonds. When hedging an existing portfolio, for instance, the investor is free to either close out the futures position before the last trading day (liquidating the hedge position), or to deliver the securities at maturity.

Where the creation of a portfolio is planned, the long position holder can decide to either take delivery of the securities when the contract is settled or alternatively, to close out the futures position and buy them on the cash market. Where there is no futures contract corresponding to the life-time of the bonds to be hedged, or if hedging individual securities in the portfolio is too complex, contracts showing a high correlation to the portfolio are used.

“Perfect Hedge” versus “Cross Hedge”

A strategy where losses incurred on changes in the value of the cash position are almost totally compensated for by changes in the value of the futures position is called a “perfect hedge”. In practice, due to the stipulation of trading integer numbers of contracts and the incongruity of cash securities and futures, a totally risk-free portfolio is not usually feasible. Frequently there is also a difference between the remaining life-time of the futures contracts and the hedging period. A “cross hedge” involves strategies where – for the reasons outlined above – the hedge position does not precisely offset the performance of the hedged portfolio.

Hedging Considerations

Basis risk – the cost of hedging

The performance of any hedge, however, does depend upon the correlation between the price movement of the underlying asset to be hedged and that of the futures or options contract used.

With government bond futures we know that the futures price will closely track the price movement of the cheapest-to-deliver (CTD) bond. Hedging with exchange traded futures has the effect of transferring outright market risk into what is termed “basis risk”. Basis risk reflects the over- or underperformance of a hedge, and is due to the nature of the hedge instrument vis-à-vis the underlying asset to be hedged.

Degree of basis risk

Hedgers are often prepared to tolerate a certain degree of basis risk in order to manage their bigger exposure to the market. Since exchanges provide very liquid and transparent marketplaces for government bond futures, it is not uncommon for certain hedgers to use these contracts to hedge non-CTD bonds which may even include corporate bonds. Naturally, the greater the disparity between the bond to be hedged and the actual CTD bond, the less reliable the hedge will be, creating in some cases significant basis risk.

CTD and non-CTD hedges

We have already seen (see chapter “Conversion Factor (Price Factor) and Cheapest-to-Deliver (CTD) Bond”) how the hedge ratio employs the use of the price factor. When hedging a CTD bond the price factor allows for a good hedge performance provided the shape of the yield curve doesn’t alter too much over time. If a change in the yield curve results in a change in the status of the CTD bond during the life of the hedge, then the success of the futures hedge may be affected. The hedger needs to monitor the situation and, if need be, alter the hedge position to reflect the changed relationship.

Determining the Hedge Ratio

The relationship of the futures position to the portfolio – in other words, the number of futures contracts required for the hedge – is referred to as the “hedge ratio”. Given the contract specifications, only integer multiples of a futures contract may be traded. Various procedures to determine the hedge ratio have been developed, providing differing degrees of precision. The following section outlines the most common procedures.

Nominal Value Method

In this method, the number of futures contracts to be used is derived from the relationship between the portfolio's nominal value and that of the futures contracts chosen for the hedge. While being the simplest of the methods described, the nominal value method is also the most imprecise in mathematical terms. The hedge ratio is determined as follows using the nominal value method:

$$\text{Hedge ratio} = \frac{\text{Nominal value of the bond portfolio}}{\text{Nominal value of fixed income futures}}$$

Nominal value of the bond portfolio = Sum of nominal value of the bonds

Nominal value of the fixed income futures = Nominal contract size of a fixed income future (CHF 100,000 or EUR 100,000)

Possible different interest rate sensitivity levels of the futures and bonds are not accounted for.

Modified Duration Method

The modified duration can be used to calculate the interest rate sensitivity of the cash and futures position, and to set the hedge ratio accordingly.

The modified duration method uses the following given factors to calculate the hedge ratio:

The cheapest-to-deliver (CTD) bond	as the underlying security of the futures ¹⁵ ;
The modified duration	of the individual positions and thus of the total portfolio, as a measure of their interest rate sensitivity. The modified duration of the portfolio corresponds to the modified duration of its component securities, weighted using the present value ¹⁶ ;
The conversion factor	which standardizes the different coupons to 6% ¹⁷ .

¹⁵ Cf. chapter "Conversion Factor (Price Factor) and Cheapest-to-Deliver (CTD) Bond".

¹⁶ Cf. chapters "Macaulay Duration" and "Modified Duration".

¹⁷ Cf. chapter "Conversion Factor (Price Factor) and Cheapest-to-Deliver (CTD) Bond".

The modified duration of the futures position is expressed as the modified duration (MD) of the cheapest-to-deliver bond, divided by the conversion factor (based on the assumption that futures price = cheapest-to-deliver/conversion factor). Using the modified duration method, the hedge ratio is calculated as follows:

$$\text{Hedge ratio} = \frac{\text{Market value of the bond portfolio}}{\text{Price (CTD)} \times 1,000} \times \frac{\text{Modified duration of the bond portfolio}}{\text{Modified duration (CTD)}} \times \text{Conversion factor}$$

This method is limited by the conditions of the duration model, as illustrated in the sections on “Macaulay Duration” and “Modified Duration”. However, the error of assessment in this calculation method is partly compensated for by the fact that both the numerator and the denominator are affected by the simplified assumptions in the hedge ratio calculation.

Motive

A pension fund manager is expecting a term deposit of CHF 10,000,000 to be repaid in mid-June. As he/she expects interest rates in all maturity sectors to fall, he/she wants to lock in the current (March) price level of the Swiss bond market.

Initial Situation

Market value of the bond portfolio	CHF 10,000,000
Price of the CTD bond	98.74
CONF Future June 2002	120.50
Modified duration of the bond portfolio	7.00%
Modified duration of the CTD	7.56%
CTD conversion factor	0.819391

Strategy

A profit should be made on the futures position by buying CONF Futures June 2002 in March at a price of 120.50 and closing out at a higher price at a later date. This should largely compensate for the forecast price increase of the planned bond purchase.

Hedge ratio using the modified duration method:

$$\text{Hedge ratio} = \frac{10,000,000}{98,740} \times \frac{7.00}{7.56} \times 0.819391 = 76.84 \text{ contracts}$$

The hedge is entered into in March by buying 77 CONF Futures contracts at a price of 120.50.

Changed Market Situation

In line with expectations, market yields have fallen by 0.30 percentage points (= 30 basis points, "bp") in June. The investor closes out the futures position.

Market value of the bond portfolio	CHF 10,210,000
Price of the CTD bond	101.01
Price CONF Future June 2002	123.27

Result

Date	Bond portfolio	CHF	CONF Future	CHF
March	Market value	10,000,000	77 contracts bought at 120.50	– 9,278,500
June	Market value	10,210,000	77 contracts sold at 123.27	+ 9,491,790
	Loss	= – 210,000	Profit	= 213,290

The overall result of the investor's position is as follows:

CHF	Profit (long CONF position)	213,290
	Loss (higher bond purchase price)	– 210,000
CHF	Result of the hedge	= 3,290

The increased investment of CHF 210,000 was more than offset by the counter position.

Sensitivity Method

The "sensitivity" or "basis point value" method is also based on the duration concept. Hence, the same assumptions prevail. However, interest rate sensitivity, as an indicator of the change in value of a security, is expressed as a one basis point (0.01 percent) interest rate change.

Using the sensitivity method, the hedge ratio is calculated as follows¹⁸:

$$\text{Hedge ratio} = \frac{\text{Basis point value of the cash position}}{\text{Basis point value of the CTD bond}} \times \text{Conversion factor}$$

$$\begin{aligned} \text{Basis point value (sensitivity) of the cash position} &= \\ \text{Market value of the bond portfolio} \times \text{MD}_{\text{bond portfolio}} / 10,000 \end{aligned}$$

$$\begin{aligned} \text{Basis point value (sensitivity) of the CTD bond} &= \\ \text{Market value of the CTD bond} \times \text{MD}_{\text{CTD}} / 10,000 \end{aligned}$$

¹⁸ The basis point value is equivalent to the modified duration, divided by 10,000, as it is defined as absolute (rather than percent) present value change per 0.01 percent (rather than 1 percent) change of market yields.

Motive

An institutional investor wants to cut down his/her bond portfolio in the coming two months. It is valued at EUR 40,000,000 as of March. The investor fears that market interest rates could rise and prices could fall until the time of the planned sale.

Initial Situation

Market value of the bond portfolio	EUR 40,000,000
Price Euro Bund Future June 2002	106.00
Price of the CTD bond	95.12
Modified duration of the bond portfolio	8.20%
Basis point value of the bond portfolio	EUR 32,800.00
Modified duration of the CTD	7.11%
CTD conversion factor	0.897383
Basis point value of the CTD bond	EUR 67.63

Strategy

A profit is made on the futures position by selling the Euro Bund Futures June 2002 at a price of 106.00 in March and buying them back cheaper at a later date. This should compensate for the price loss incurred on the bonds.

Hedge ratio using the basis point value method:

$$\text{Hedge ratio} = \frac{\text{Basis point value of the bond portfolio}}{\text{Basis point value of the CTD bond}} \times \text{Conversion factor}$$

$$\text{Hedge ratio} = \frac{32,800.00}{67.63} \times 0.897383 = 435.22 \text{ contracts}$$

The position can be hedged by selling 435 Euro Bund Futures June 2002 in March, at a price of 106.00.

Changed Market Situation

Market yields have risen by 0.30 percentage points (= 30 basis points, bp) in June.

The investor closes out the short futures position by buying back the Euro Bund Futures June 2002.

Market value of the bond portfolio:	EUR 39,016,000
Market value of the CTD bond:	EUR 93,091.10
Euro Bund Future June 2002:	103.73

Result

Date	Bond portfolio	EUR	Euro Bund Future	EUR
March	Market value	40,000,000	435 contracts sold at 106.00	46,110,000
June	Market value	39,016,000	435 contracts bought at 103.73	– 45,122,550
	Loss	= – 984,000	Profit	= 987,450

The overall result of the investor's position is as follows:

EUR	Profit (short Euro Bund position)	987,450
EUR	Loss (loss in value of the portfolio)	– 984,000
EUR	Result of the hedge	= 3,450

The profit made on the Euro Bund Futures position more than compensates for the loss incurred on the bond portfolio.

Static and Dynamic Hedging

Simplifying the interest rate structure upon which the hedging models are based can, over time, lead to inaccuracies in the hedge ratio. Hence, it is necessary to adjust the futures position to ensure the desired total or partial hedge. This kind of continuous adjustment is called “dynamic” hedging or “tailing”. In contrast, “static” hedging means that the original hedge ratio remains unchanged during the life of the hedge. Investors must weigh up the costs and benefits of an adjustment.

Cash-and-Carry Arbitrage

In principle, arbitrage is defined as creating risk-free (closed) positions by exploiting mispricing in derivatives or securities between two market places. The so-called “cash-and-carry” arbitrage involves the purchase of bonds on the cash market and the sale of the relevant futures contracts. Selling bonds and simultaneously buying futures is referred to as a “reverse cash-and-carry” arbitrage. In each case, the investor enters into a long position in an undervalued market (cash or futures). Although these arbitrage transactions are often referred to as “risk-free”, their exact result depends on a variety of factors, some of which may in fact hold certain risks. These factors include the path of price developments and the resulting variation margin flows, as well as changes in the CTD during the term of the transaction. A detailed review of all influences on cash-and-carry/reverse cash-and-carry positions would exceed the scope of this brochure. The theoretically correct basis can be determined by discounting the delivery price. Cash-and-carry opportunities tend to arise for very short periods only and rarely exceeds the transaction costs incurred.

The principle of cash-and-carry arbitrage is explained below, using the valuation example taken from the section on futures pricing.

Starting Scenario

Valuation date	May 8, 2002 ("today")
CTD bond	3.75% Federal Republic of Germany debt security due on January 4, 2011
Price of the CTD bond	90.50
Accrued interest	$3.75\% \times 124/365 \times 100 = 1.274\%$
Conversion factor	0.852420
Money market rate	3.63%
Theoretical futures price ¹⁹	106.12
Traded futures price	106.46
Delivery date of the future	June 10, 2002

If the futures contract trades above its theoretically correct price, the arbitrageur buys deliverable bonds and enters into a short position in the relevant futures contract. The arbitrageur carries out the following transactions, based on an individual futures contract:

Transaction	EUR	Remarks
CTD bought	91,774.00	Clean price 90,500 + 1,274 accrued interest
Financing costs until futures maturity	301.19	$91,774.00 \times 0.0363 \times (33/365)$ years
Total amount invested in the bonds	92,075.19	$91,774.00 + 301.19$

This total investment must be compared to the delivery price at the futures' maturity, in addition to the profit and loss settlement during the lifetime. The delivery price is derived from the final settlement price, multiplied by the conversion factor plus the accrued interest.

Transaction	EUR	Remarks
Futures sold	106,460.00	Value on May 4
Final settlement price	106,350.00	
Profit from variation margin	110.00	
delivery price	92,267.87	$106,350 \times 0.852420 + 1,613.00$ accrued interest

If the profit made on the short position is added as income to the delivery price, the profit made on the arbitrage transaction is equivalent to the difference to the capital invested.

Total profit:	$92,267.87 + 110.00 - 92,075.19 = 302.68$
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The price imbalance has resulted in a profit of EUR 302.68 for the investor.

¹⁹ Cf. chapter „Conversion Factor and Cheapest-to-Deliver Bond“.

Introduction to Options on Fixed Income Futures

Options on Fixed Income Futures – Definition

An option is a contract entered into between two parties. By paying the option price (the premium) the buyer of an option acquires the right, for example,

... to buy	Call option	Calls
... or to sell	Put option	Puts
... a given fixed income future	Underlying instrument	Euro Bund Future
... in a set amount	Contract size	One contract
... for a set exercise period	Expiration	May 24, 2002
... at a determined price	Exercise price	106.50

If the buyer claims his/her right to exercise the option, the seller is obliged to sell (call) or to buy (put) the futures contract at a set exercise price. The option buyer pays the option price (premium) in exchange for this right. This premium is settled according to the futures-style method. In other words the premium is not fully paid until the option expires or is exercised. This means that, as with futures, the daily settlement of profits and losses on the option premium is carried out via variation margin (cf. Chapter "Variation Margin").

Options on Fixed Income Futures – Rights and Obligations

Investors may assume a position in the option market by buying or selling options:

Long position	Short position
An investor buying options assumes a long position. This can be a long call or a long put position, depending on the contract.	An investor selling an option assumes a short position. This can be a short call or a short put position, depending on the contract.

Buyers and sellers of options on fixed income futures have the following rights and obligations:

Call option		Put option	
Call buyer	Call seller	Put buyer	Put seller
Long call	Short call	Long put	Short put
The buyer of a call has the right, but not the obligation, to buy the futures contract at an exercise price agreed in advance.	In the event of exercise, the seller of a call is obliged to sell the futures contract at an exercise price agreed in advance.	The buyer of a put has the right, but not the obligation, to sell the futures contract at an exercise price agreed in advance.	In the event of exercise, the seller of a put is obliged to buy the futures contract at an exercise price agreed in advance.

An option position on fixed income futures can be liquidated by closing it out (see below); the buyer of the option can also close it by exercising the option.

Closeout

Closing out means neutralizing a position by a counter transaction. In other words, a short position of 2,000 Euro Bund Futures June 2002 calls with an exercise price of 104.50 can be closed out by buying 2,000 call options of the same series. In this way, the seller's obligations arising from the original short position have lapsed. Accordingly, a long position of 2,000 Euro Bund Futures June 2002 puts with an exercise price of 104.50 can be closed out by selling 2,000 put options of the same series.

Exercising Options on Fixed Income Futures

If an option on a fixed income future is exercised by the holder of the long position, the clearinghouse matches this exercise with an existing short position, using a random process. This is referred to as an "assignment" of the short position. When this happens, the options are dissolved and the buyer and seller enter into the corresponding futures positions. The decisive factor is the option's exercise price, which is applicable as the purchase or sale price of the futures position. The corresponding futures positions opened according to the original options position are outlined in the following table:

Exercising a ...		Assignment of a ...	
long call option	long put option	short call option	short put option
leads to the following position being opened ...			
long futures position	short futures position	short futures position	long futures position

Options on fixed income futures can be exercised on any exchange trading day until expiration (American-style options). The option expiration date is prior to the last trading day of the futures contract. An option holder wishing to exercise his/her right must inform the clearing house, which in turn nominates a short position holder by means of a neutral random assignment procedure.

Contract Specifications – Options on Fixed Income Futures

Eurex options are exchange-traded contracts with standardized specifications. The Eurex product specifications are set out on the Eurex website (www.eurexchange.com) and in the “Eurex Products” brochure.

The most important terms are described in the following example.

An investor buys:

...20	Options on Euro Bund Futures	One contract comprises the right to buy or sell a single futures contract.
... June 2002	Expiration date	Each option has a limited lifetime and a set expiration date. The expiration months available for trading are the three nearest calendar months, as well as the following month within the March, June, September and December cycle; i.e. lifetimes of one, two and three months, as well as a maximum of six months are available. Hence, for the months March, June, September and December, the expiration months for the option and the maturity months for the underlying futures are identical (although the last trading days differ for options and futures). In the case of the other contract months, the maturity month of the underlying instrument is the quarterly month following the expiration date of the option. Hence, the option always expires before the maturity of the underlying futures contract.
... 106.50	Exercise price (also called “strike price”)	This is the price at which the buyer can enter into the corresponding futures position. At least nine exercise prices per contract month are always available. The price intervals of this contract are set at 0.50 points.
...Call	Call option	The buyer can convert this position into a long futures position. Upon exercise the seller enters into a short position.
... Options on the Euro Bund Future	Underlying instrument	The Euro Bund Future is the underlying instrument for the option contract.
... at 0.15	Option price (premium)	Buyers of options on fixed income futures pay the option price to the seller upon exercise, in exchange for the right. The option premium is EUR 10.00 per 0.01 points. Therefore a premium of 0.15 is really worth EUR 150. The premium for 20 contracts is $20 \times \text{EUR } 150 = \text{EUR } 3,000$.

In our example, the buyer acquires the right to enter into a long position of 20 Euro Bund Futures, at an exercise price of 106.50, and pays EUR 3,000 to the seller in exchange. Upon exercise, the seller of the option is obliged to sell 20 Euro Bund Futures June 2002 contracts at a price of 106.50. This obligation is valid until the option's expiration date.

Premium Payment and Risk Based Margining

Contrary to equity or index options, buyers of options on fixed income futures do not pay the premium the day after buying the contract. Instead the premium is paid upon exercise, or at expiration. Contract price changes during the lifetime are posted via variation margin. When the option is exercised, the buyer pays the premium to the value of the daily settlement price on this day. This method of daily profit and loss settlement is called "futures-style premium posting". Additional margin equivalent to that for the underlying future has to be deposited to cover the price risk.

Rationale

The investor expects the price of the Euro Bund Future June 2002 to fall, and decides to enter into a put option position. In this way, the risk will be limited if the expectation should turn out to be wrong.

Strategy

On May 13, the Euro Bund Future June 2002 is trading at 150.78. The investor buys 10 put options on this contract with an exercise price of 106.00. The price is 0.55 points, which equals EUR 550 per option contract.

Date	Transaction	Purchase/ selling in EUR	Option daily settlement price in EUR	Variation margin ²⁰ credit in EUR	Variation margin debit in EUR	Additional margin ^{21, 22} in EUR
05/13	10 Put options bought	0.55	0.91	3,600		16,000
05/14			0.81		-1,000	

Changed Market Situation

The Euro Bund Future is now trading at 105.50. The investor decides to exercise the option, which is trading at 0.70.

05/15	Exercise	0.70			-3,100	
05/16	Opening of a short position in the Euro Bund Future June 2002					+/-0 In this case, the futures additional margin rates cor- respond to the option
	Total up to entry into futures position			3,600	-4,100	

²⁰ Cf. chapter "Variation Margin".

²¹ Cf. chapter "Futures Spread Margin and Additional Margin".

²² The Additional Margin may vary because of changes in volatility or time value decay.

The variation margin on the exercise day (05/15) is calculated as follows:

Profit made on the exercise	EUR 5,000	Difference between the exercise price (106.00) and the daily settlement price (105.50) multiplied by the contract value and the number of contracts.
Change in value of the option compared to the previous day	EUR -1,100	EUR 7,000 – EUR 8,100
Option premium to be paid	EUR -7,000	$0.70 \times 10 \times 1,000$
Variation margin on 05/15/2002	EUR -3,100	

Result of the Exercise

These transactions result in a total loss of EUR -500 for the investor. This loss can be expressed either as the difference between the option price of EUR 5,500 ($0.55 \times 10 \times \text{EUR } 1,000$), which was fixed when the agreement was concluded but which was not paid in full until exercise, and the profit of EUR 5,000 made from exercising; or as the net balance of variation margin flows (EUR 3,600 – EUR 4,100). When an option is exercised, the change in value of the option between the time of purchase and entering into the futures position has no impact on the investor's end result. Additional margin for options on the Euro Bund Future is equal to that of the underlying instrument, so that no further margin call is required should the position be exercised.

In this case, however, it would not be appropriate to exercise the option, since the investor can make a profit above the initial purchase price if he/she closes out, i.e. sells the put option.

Result of Closeout

Given the previous day's settlement price of 0.81, a sale on May 15 at a price of 0.70 also prevailing during the day would only result in a variation margin debit of EUR 1,100 ($0.11 \times 10 \times \text{EUR } 1,000$). The profit and loss calculation for the sale of the option is shown below:

Date	Transaction	Purchase/ selling in EUR	Option daily settlement price in EUR	Variation margin profit in EUR	Variation margin loss in EUR	Additional margin in EUR
[...]	[...]	[...]	[...]	[...]	[...]	
05/15	Sale	0.70			-1,100	
05/16						-16,000
Total				3,600	-2,100	

By selling the options, the investor makes a total profit of EUR 1,500. This is derived from the difference between the selling and purchase price ($0.70 - 0.55$), multiplied by the contract value and the number of futures contracts. Additional margin is released to the investor.

Options on Fixed Income Futures – Overview

The following three option contracts on fixed income futures are currently traded at Eurex:

Products	Product code
Option on the Euro Schatz Future	OGBS
Option on the Euro Bobl Future	OGBM
Option on the Euro Bund Future	OGBL

Option Price

Components

The option price is comprised of two components – intrinsic value and time value.

$$\text{Option value} = \text{Intrinsic value} + \text{Time value}$$

Intrinsic Value

The intrinsic value of an option on fixed income futures corresponds to the difference between the current futures price and the option's exercise price, as long as this difference represents a price advantage for the option buyer. Otherwise, the intrinsic value equals zero.

For calls: Intrinsic value = Futures price – Exercise price of the option, if this is > 0; otherwise it is zero.
For puts: Intrinsic value = Exercise price – Futures price, if this is > 0; otherwise it is zero.

An option is in-the-money, at-the-money or out-of-the-money depending upon whether the price of the underlying is above, at, or below the exercise price.

	Calls	Puts
Exercise price < Futures price	in-the-money (intrinsic value > 0)	out-of-the-money (intrinsic value = 0)
Exercise price = Futures price	at-the-money (intrinsic value = 0)	at-the-money (intrinsic value = 0)
Exercise price > Futures price	out-of-the-money (intrinsic value = 0)	in-the-money (intrinsic value > 0)

An option always has intrinsic value (is always in-the-money) if it allows the purchase or sale of the underlying instrument at better conditions than those prevailing in the market. Intrinsic value is never negative, as the holder of the option is not obliged to exercise the option.

Time Value

Time value reflects the buyer's chances of his/her forecasts on the development of the underlying instrument during the remaining lifetime being met. The buyer is prepared to pay a certain sum – the time value – for this opportunity. The closer an option moves towards expiration, the lower the time value becomes until it eventually reaches zero on that date. The time value decay accelerates as the expiration date comes closer.

$$\text{Time value} = \text{Option price} - \text{Intrinsic value}$$

Determining Factors

The theoretical price of options on fixed income futures can be calculated using different parameters, regardless of the current supply and demand scenario. An important component of the option price is the intrinsic value as introduced earlier (cf. section on “Intrinsic Value”). The lower (call) or higher (put) the exercise price compared to the current price, the higher the intrinsic value and hence the higher the option price. An at-the-money or out-of-the-money option comprises only time value. The following section illustrates the determining factors of time value.

Volatility of the Underlying Instrument

Volatility measures the propensity of price fluctuations in the underlying instrument. The greater the volatility, the higher the option price. Since an underlying instrument which is subject to strong price fluctuations provides option buyers with a greater chance of meeting their price forecast during the lifetime of the option, they are prepared to pay a higher price for the option. Likewise, sellers demand a higher return to cover their increasing risks.

There are two types of volatility:

Historical volatility	Implied volatility
This is based on historical data and represents the standard deviation of the returns of the underlying instrument.	This corresponds to the volatility reflected in a current market option price. In a liquid market it is the indicator for the changes in market yields expected by the market participants.

Remaining Lifetime of the Option

The longer the remaining lifetime, the greater the chance that the forecasts of option buyers on the price of the underlying instrument will be met during the remaining time. Conversely, the longer the lifetime the higher the risk for the option seller, calling for a higher price for the option. The closer it moves towards expiration, the lower the time value and hence the lower the option price *ceteris paribus*. As the time value equals zero on the expiration date, the course of time acts against the option buyer and in favour of the option seller.

The time value is relinquished when the option is exercised. This generally minimizes the investor's earnings (cf. chapter “Premium Payment and Risk Based Margining”).

Influencing Factors

The price of the call is higher,	The price of the call is lower,
the higher the price of the underlying instrument;	the lower the price of the underlying instrument;
the lower the exercise price;	the higher the exercise price;
the longer the remaining lifetime;	the shorter the remaining lifetime;
the higher the volatility.	the lower the volatility.

The price of the put is higher,	The price of the put is lower,
the lower the price of the underlying instrument;	the higher the price of the underlying instrument;
the higher the exercise price;	the lower the exercise price;
the longer the remaining lifetime;	the shorter the remaining lifetime;
the higher the volatility.	the lower the volatility.

Important Risk Parameters – “Greeks”

An option's price is affected by a number of parameters, principally changes in the underlying price, time and volatility. In order to estimate the changes in an options price, a series of sensitivities are used, which are known as the “Greeks”.

The price calculations in this chapter are based on the assumption that the only changes that occur are those that are given and that all other influencing factors remain constant (“ceteris-paribus” assumption).

Delta

The delta of an option indicates the change in an options price for a one unit change in the price of the underlying futures contract. The delta changes according to fluctuations in the underlying instrument. With calls, the delta lies between zero and one. It lies between minus one and zero with puts.

Call option deltas	$0 \leq \text{delta} \leq 1$
Put option deltas	$-1 \leq \text{delta} \leq 0$

The value of the delta depends on whether an option is in-, at- or out-of-the-money:

		Out-of-the-money	At-the-money	In-the-money
Long	Call	$0 < \text{delta} < 0.50$	0.50	$0.50 < \text{delta} < 1$
	Put	$-0.50 < \text{delta} < 0$	-0.50	$-1 < \text{delta} < -0.50$
Short	Call	$-0.50 < \text{delta} < 0$	-0.50	$-1 < \text{delta} < -0.50$
	Put	$0 < \text{delta} < 0.50$	0.50	$0.50 < \text{delta} < 1$

The delta can be used to calculate option price changes. This is shown in the following example:

Initial Situation

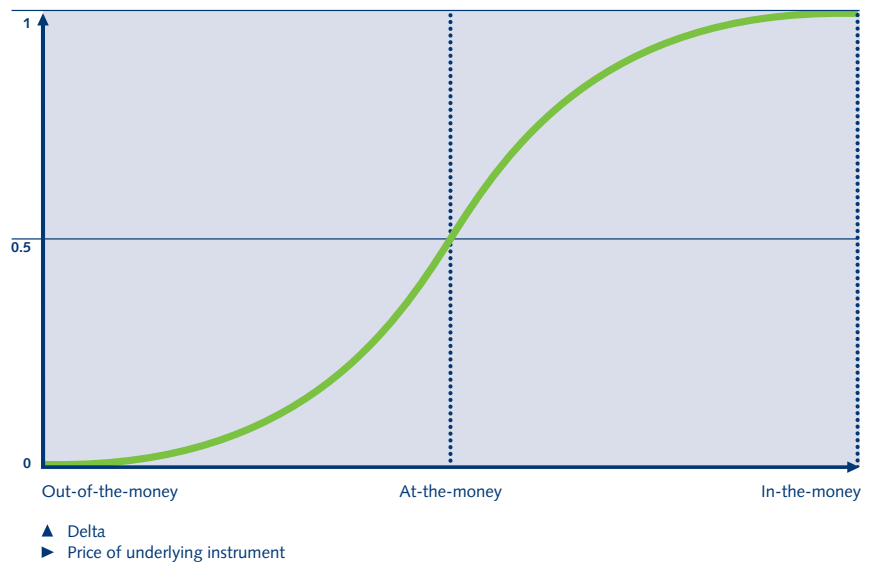
Call option on the Euro Bund Future June 2002	104.75	0.97
Call delta		0.69
Euro Bund Future June 2002		105.50

Using the delta to calculate the value of the call option as a result of price changes in the underlying instrument:

Changes in the futures price			Changes in the price of the call on the futures		
Price	Price change	New price	Price	Price change according to delta	New price
105.50	+ 1.00	106.50	0.97	+ 0.69 (= 1×0.69)	1.66
106.50	- 0.50	106.00	1.66	- 0.345 (0.5×0.69)	1.315

The dependency of the option price on price changes in the underlying futures contract is displayed in the following chart:

Correlation between Delta of an Option and Price Changes in the Underlying Instrument



Gamma

As the underlying futures price changes, so too does the delta of an option. Gamma can be described as the rate of change of delta. The greater the gamma, the greater the reaction of the delta to price changes in the underlying instrument. Gamma can thus be used to recalculate delta. With long option positions the gamma factor is always positive. The gamma is at its highest level for at-the-money options immediately before expiration.

Initial Situation

Call option on the Euro Bund Future June 2002	106.00	0.49 (= EUR 490)
Call delta		0.46
Gamma		0.2890
Euro Bund Future June 2002		105.85

Changed Market Situation

Price change in the Euro Bund Future		
from 105.85	by 0.10	to 105.95

Using the delta factor (old) to recalculate the option price		
from 0.49	by 0.046	to 0.54 (rounded)
or from EUR 490	by $0.046 \times \text{EUR } 1,000$	to EUR 540 (rounded)

Using the gamma factor to recalculate the delta factor		
from 0.46	by 0.02890	to 0.4889

If the price of the underlying instrument increases by an additional 0.10 percentage points, from 105.95 to 106.05, the new delta factor can be used to calculate the change in the option price.

Using the delta factor (new) to recalculate the option price		
from 0.54	by 0.04889	to 0.59 (rounded)
or from EUR 540	by $0.04889 \times \text{EUR } 1,000$	to EUR 590

Vega (Kappa)

Vega is a measure of the impact of volatility on the option price. Vega indicates by how many units the option price will change given a one percentage point change in the expected volatility of the underlying instrument. The longer the remaining lifetime of the option, the higher the vega. It is at its maximum with at-the-money options and shows identical behavior for both calls and puts.

The following example outlines how the option price reacts to a change in volatility.

Initial Situation

Call option on the Euro Bund Future June 2002	0.49 (= EUR 490)
Expected volatility	5.80%
Call vega	9.41

Changed Market Situation

Change in volatility		
from 5.80%	by 1%	to 6.80%

Changes According to the New Market Situation

Using the vega to recalculate the option price		
from 0.49	by 0.0941	to 0.5841
or from EUR 490	by $0.0941 \times \text{EUR } 1,000$	to EUR 584.10

Theta

Theta describes the influence of the time value decay on the option price. Theta indicates by how many units the option price will change given a one period reduction in the remaining lifetime. Its value is defined as the derivative of the option price for the remaining lifetime multiplied by minus one. With long positions in options on fixed income futures, its value is always negative. This effect is called time value decay or time decay. As options near expiration, time value decay increases in intensity. The decay is at its maximum with at-the-money options immediately before expiration.

Trading Strategies for Options on Fixed Income Futures

Options on fixed income futures can be used to implement strategies to exploit price changes in the respective fixed income futures contract and its underlying instruments, while limiting the exposure to the option premium paid. These strategies therefore combine the motive for trading fixed income futures with the risk and reward profile of options. As previously mentioned, the underlying instruments available for trading are the Euro Bund, Euro Bobl and the Euro Schatz Futures.

The four basic options positions, including spreads and synthetic positions are outlined in this section.

Long Call

Rationale

The investor wants to benefit from an expected price rise in fixed income futures, while limiting potential losses in the event of his/her forecast being inaccurate.

Initial Situation

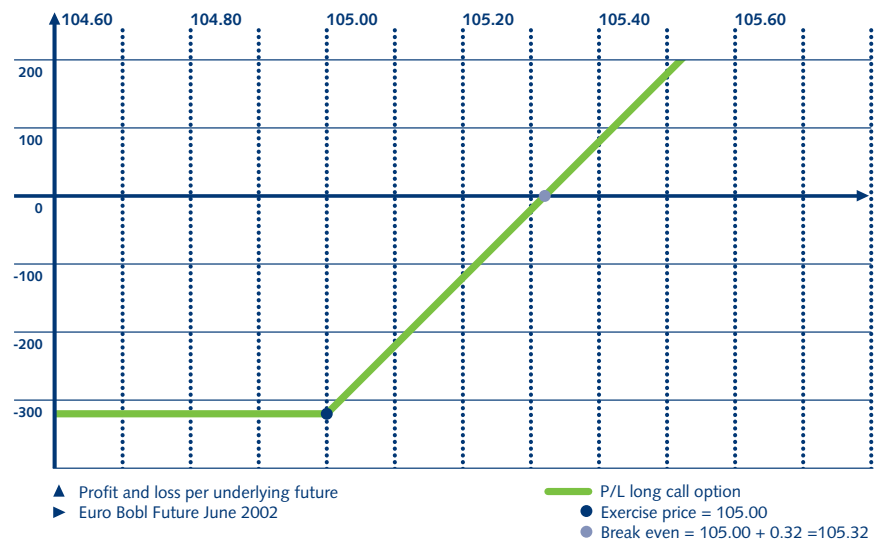
Euro Bobl Future June 2002		104.98
Call option on the Euro Bobl Future June 2002	105.00	0.32

The investor buys 20 contracts of the 105.00 call on the Euro Bobl Future June 2002 at a price of 0.32.

Changed Market Situation

The futures price has risen to 105.27 at the beginning of June. The option is trading at 0.35. Although exercising the option would allow the investor to enter into a long futures position at 27 ticks below the market price (or EUR 270 per contract = intrinsic value), this would be less than the option premium of 0.32 paid. Moreover, an exercise would forego the time value of 0.08 (premium less intrinsic value). However by selling the option, the investor can take the profit on the price increase from 0.32 to 0.35 ($20 \times \text{EUR } 30 = \text{EUR } 600$). The profit and loss profile of the long call option is illustrated in the following diagram. Note that the analysis is based on expiration; time value is therefore not taken into account.

**Profit and Loss Profile on the Last Trading Day, Long Call Option on the Euro Bobl Future
June 2002 – P/L in EUR**



Short Call

Rationale

The investor expects 5-year yields on the German capital market to remain unchanged, or to rise slightly. Based on this forecast, he/she expects the price of the Euro Bobl Future to remain constant or fall slightly.

Initial Situation

The investor does not hold any long futures position.

Euro Bobl Future June 2002		104.92
Call option on the Euro Bobl Future June 2002	105.00	0.32

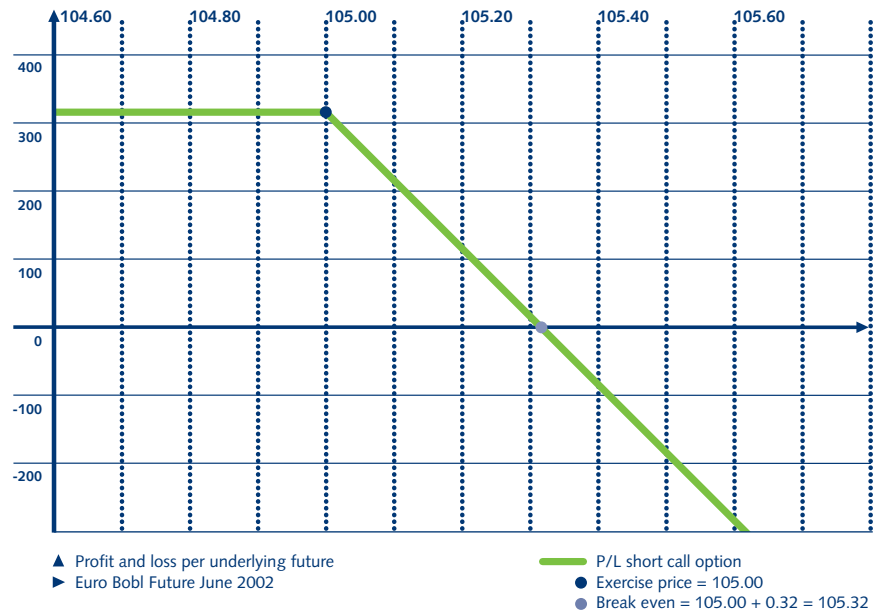
Strategy

The investor sells call options on the Euro Bobl Future June 2002 at a price of 0.32, equivalent to EUR 320 per contract.

Changed Market Situation

If his/her forecast on the price development turns out to be correct, the option expires worthless and the seller makes a profit to the value of the premium received. If, however, contrary to expectations, the prices rise, the investor must expect the option to be exercised. This can be avoided by buying back the option at a higher price, thus liquidating the position. The risk exposure for such a 'naked' short call position is significant, as illustrated in the following chart showing the risk/reward profile of short call positions at expiration.

**Profit and Loss Profile on the Last Trading Day, Short Call Option on the Euro Bobl Future
June 2002 – P/L in EUR**



Long Put

Rationale

An investor expects prices of 2-year German bonds to fall. At the same time, he/she wants to limit the risk exposure of his/her position. The maximum loss of a bought option corresponds to the premium paid.

Initial Situation

Euro Schatz Future June 2002	102.82
Put option on the Euro Schatz Future June 2002	102.75 0.12

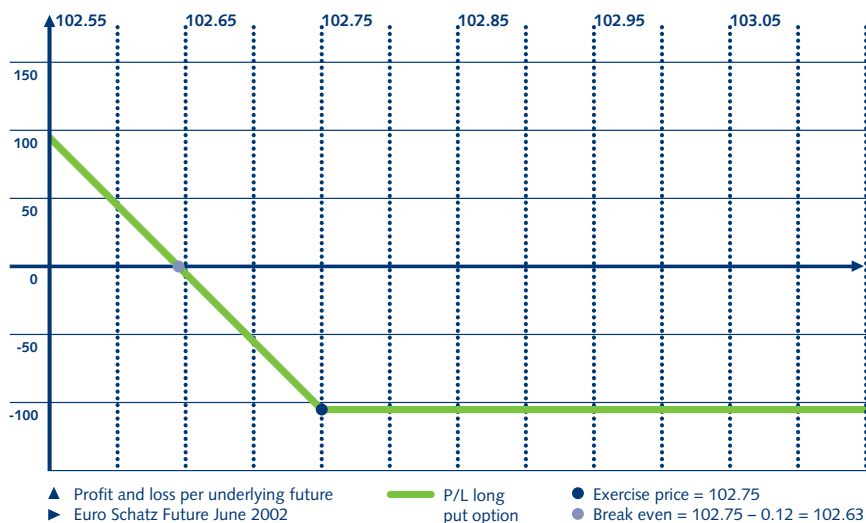
Strategy

On May 15, the investor decides to buy a put option on the Euro Schatz Future.

Changed Market Situation

Two days later, the Euro Schatz Future is trading at 102.55 and the value of the put option has risen to 0.23. The option's intrinsic value is 0.20 ($102.75 - 102.55$). At this point in time, the investor has the choice of holding, selling or exercising the option. As with the long call, exercising the option would not make sense at this point, as this would be equivalent to giving up time value of 0.03 ($0.23 - 0.20$). Instead if the option is closed out, the investor can make a profit of 0.11 per contract ($0.23 - 0.12$). If, however, the option position is held and the futures price rises, the option will be out-of-the-money and will thereby fall in value. Unless any turnaround is expected, the holder of the option will close out his/her contracts, thus avoiding a loss on the remaining time value until expiration.

Profit and Loss Profile on the Last Trading Day, Long Put Option on the Euro Schatz Future June 2002 – P/L in EUR



Short Put

Rationale

An investor expects the prices of the Euro Bund Future to stagnate or potentially to rise slightly, and is prepared to accept the risks in the event of the market going the other way.

Initial Situation

Euro Bund Future June 2002	106.00
Put option on the Euro Bund Future June 2002	106.00 0.39

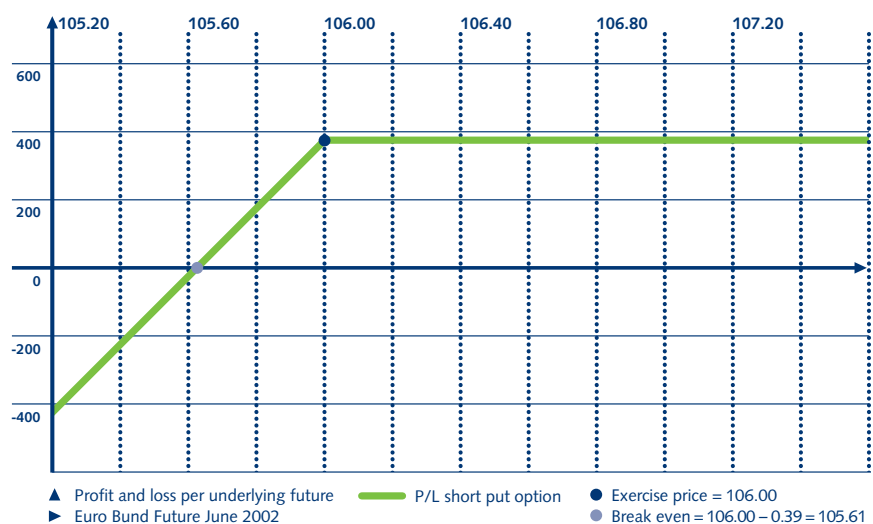
Strategy

On May 15, the investor sells put options on the Euro Bund Future at a price of 0.39.

Changed Market Situation

Two days after selling the options, the price of the Euro Bund Future has fallen to 105.63. This has pushed up the put option to 0.57. Since the short put option is now making a loss and the investor wants to avoid any further losses, he/she decides to buy back the options at the current price, limiting the loss on the short position. He/she takes a loss of 0.18 (EUR 180) per contract.

Profit and Loss Profile on the Last Trading Day, Short Put Option on the Euro Bund Future June 2002 – P/L in EUR



Bull Call Spread

Rationale

The investor expects a slight rise in the price of the Euro Bund Future. He/she wants to simultaneously limit the risk and reduce the costs of the position.

Initial Situation

Euro Bund Future June 2002		106.00
Call option on the Euro Bund Future June 2002	106.00	0.39
Call option on the Euro Bund Future June 2002	106.50	0.20

Strategy

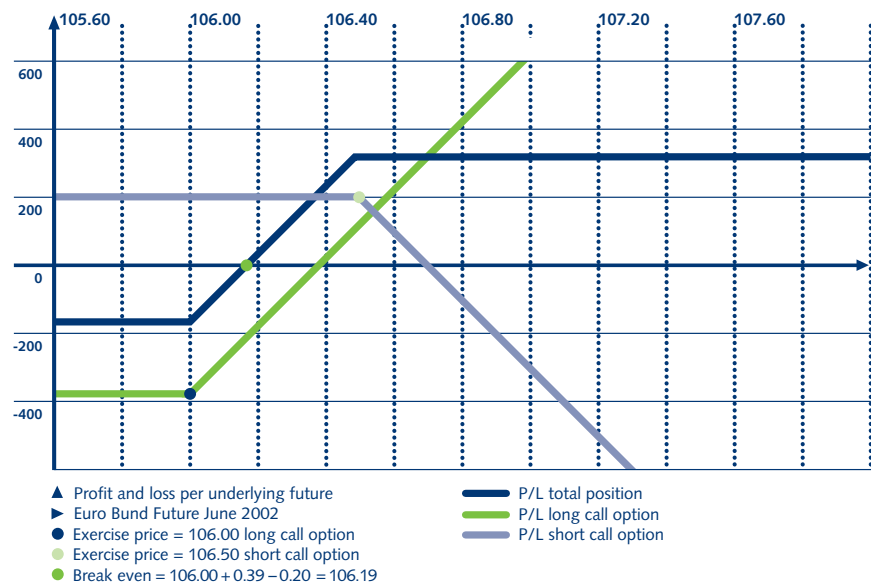
The investor decides to construct a bull call spread. This position comprises the simultaneous purchase of a call option with a lower exercise price and sale of a call option with a higher exercise price. Selling the higher exercise call puts a cap on the maximum profit, but partially covers the costs of buying the call option with a lower exercise price and thus reducing the overall strategy cost. A net investment of 0.19 points or EUR 190 per contract is required to buy the bull call spread.

Changed Market Situation

The price level of the Euro Bund Future has risen to 106.62 two weeks after opening the position. The two options are trading at 0.79 (exercise price 106.00) and 0.45 (exercise price 106.50), respectively. At this point, the investor closes out the spread and receives a net premium of 0.34 per contract. This results in a net profit of 0.15.

The following profit and loss profile is the result of holding the options until expiration.

Profit and Loss Profile on the Last Trading Day, Bull Call Spread, Option on the Euro Bund Future June 2002 – P/L in EUR



On the last trading day, the maximum profit is made when the price of the underlying instrument is equal to or lies above the higher exercise price. In this case, the profit made is the difference between the exercise prices less the net premium paid. If the price of the underlying instrument is higher, any additional profit made from the more expensive option is offset by the equivalent loss incurred on the short position.

Bear Put Spread

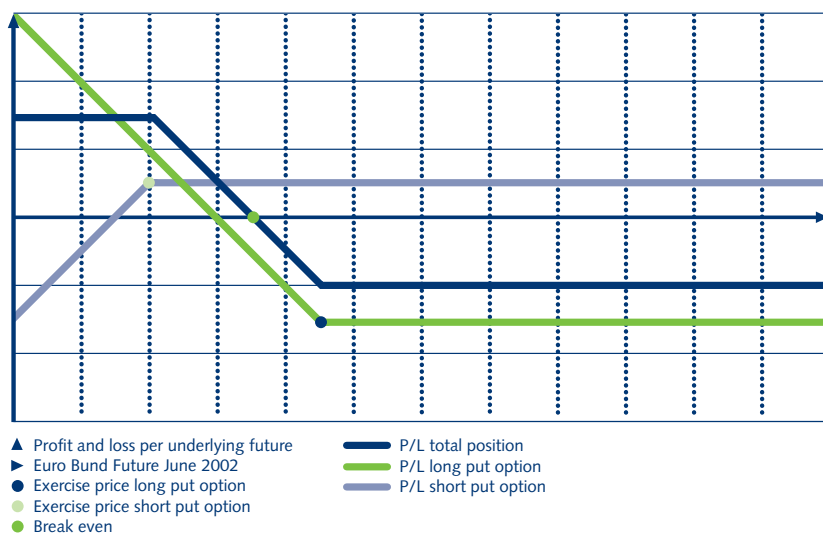
Rationale

The investor expects a slight fall in the price of the Euro Bund Future. In line with the long bull call spread, he/she wants to benefit from the expected development, but with limited investment and limited risk.

Strategy

He/she decides to construct a bear put spread by simultaneously buying a put with a higher exercise price and selling a put with a lower exercise price. The maximum loss is limited to the net premium paid. This would be incurred if the price rose to at least the level of the higher exercise price. The maximum profit, which is equivalent to the difference of the exercise prices less the net premium paid, is made if the price of the Euro Bund Future at expiration falls below the lower exercise price.

Profit and Loss Profile on the Last Trading Day, Bear Put Spread



Long Straddle

Rationale

Having remained stable for quite some time, prices of German Federal Debt Obligations (Bundessobligationen) are expected to become more volatile, although the exact market direction is uncertain.

Initial Situation

Euro Bobl Future June 2002		105.10
Call option on the Euro Bobl Future June 2002	105.00	0.27
Put option on the Euro Bobl Future June 2002	105.00	0.17

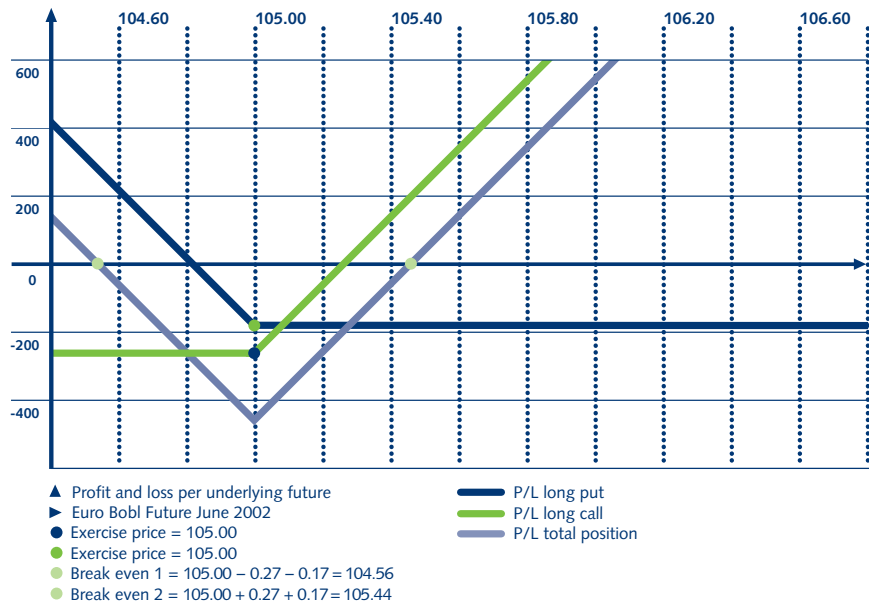
Strategy

The investor buys one at-the-money call option and one at-the-money put option, to benefit from a rise in the option prices should volatility increase. The success of his/her strategy does not necessarily depend on whether Euro Bobl Futures prices are rising or falling.

Changed Market Situation

After a period of strong price fluctuation, the Euro Bobl Future is trading at 105.35. The call option is now valued at 0.64 points, the put at 0.12. The strategy has turned out to be successful, as the volatility has resulted in a significant increase in the time value of the call option (from 0.17 to 0.29), whereas the now out-of-the-money put option has fallen only marginally (from 0.17 to 0.12). Moreover, the intrinsic value of the call option increased (from 0.10 to 0.35) while the put option shows none. It is important to note that, in the event of a price decrease or a temporary recovery in the futures price, the aggregate value of both options would have increased provided that volatility had risen sufficiently. As a double long position, a straddle is exposed to particularly strong time decay, which can offset any positive performance. For the strategy to be profitable at expiration, the price of the underlying instrument must differ from the exercise price by at least the aggregate option premium.

**Profit and Loss Profile on the Last Trading Day, Long Straddle, Option on the Euro Bobl Future
June 2002 – P/L in EUR**



Long Strangle

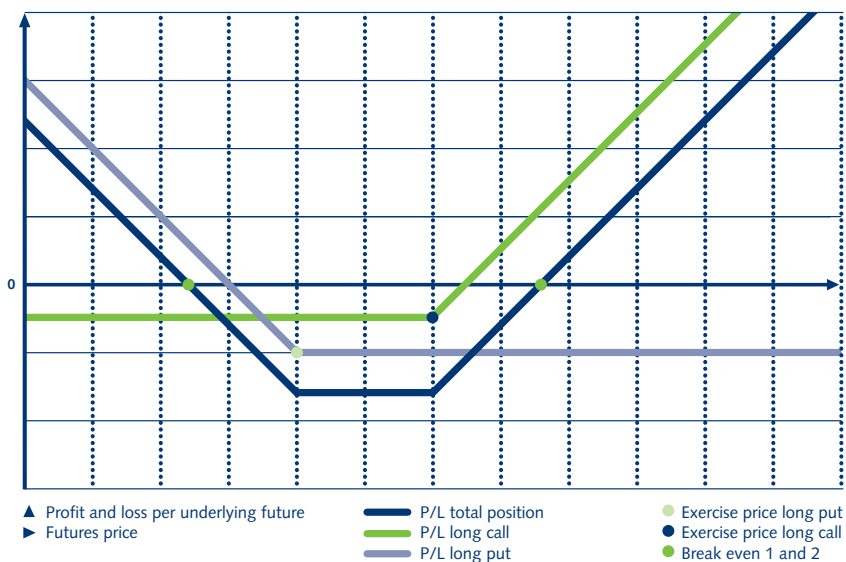
Rationale

The investor expects a significant increase in volatility in the 2-year sector of the German capital market. He/she wants to benefit from the expected development but strictly limit his/her risk exposure.

Strategy

The investor decides to buy a strangle using options on the Euro Schatz Future. Similar to the straddle, this position is made up of a long call and a long put option. With the strangle, the put has a lower exercise price than the call. The sum of the premiums and, in this case, the maximum loss, is lower than for the straddle. However, by "separating" the exercise prices, the profit potential is also reduced.

Profit and Loss Profile on the Last Trading Day, Long Strangle

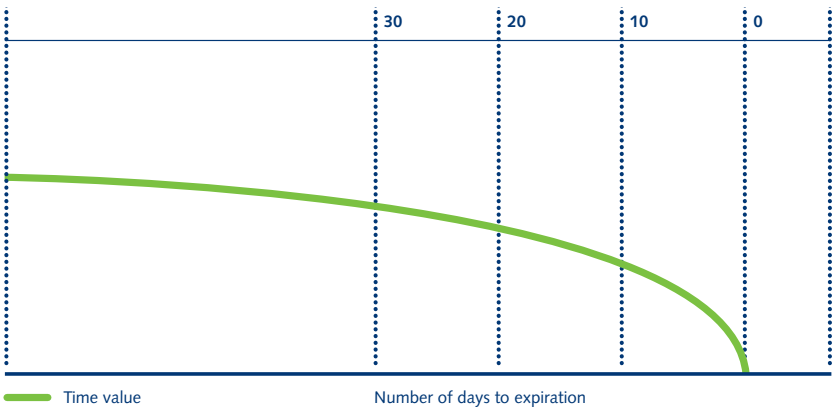


Impact of Time Value Decay and Volatility

Time Value Decay

The remaining lifetime of the options contract influences the level and the further trend of time value. As explained under “Theta”, time value declines progressively as expiration date approaches. The time decay per period is smaller for long running options than for those which are about to expire. Other things being equal, an option with a longer remaining lifetime has a higher time value and is therefore more expensive.

Time Value for Long Option Position (at-the-money)



Exercise, hold or close

Most examples are based on the assumption that an option is held until expiration. However, closing the position prior to expiration, or even exercising it during its lifetime are valid alternatives.

It is, however, unwise to exercise an option during its lifetime as the buyer forfeits time value by doing so.

After buying the option, the holder of a long call position therefore must decide whether or not to close it before the last trading day.

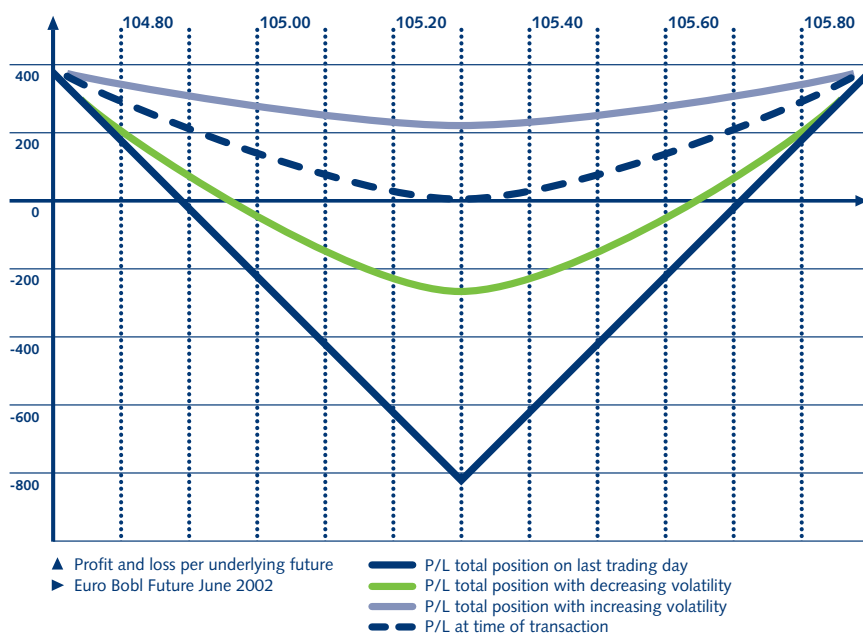
Sale of the option on the Exchange (closeout)	Exercise of the option
The profit or loss arises from the difference between the original purchase price and the current selling price (intrinsic value plus time value) of the option.	The profit arises from the difference between the intrinsic value and the price paid for the option.

Investors must continuously check during the lifetime of an option whether, according to their assessment, the further price trend will compensate for continued time decay. With a long call option, for example, the position should be closed as soon as no further rise in the share price is expected. These remarks are made on the assumption that other parameters, in particular volatility, remain constant.

Impact of Fluctuations in Market Volatility

The example covering the purchase of a straddle which was covered before was based on the assumption that the position was held until expiration of the options. At that point in time, a profit will only be made if the share price deviates from the exercise price by more than the sum of the two option premiums. In practice, however, it is rather unlikely that there will be a profit on the last trading day, due to the dual loss of time value. Rather, often the aim of this strategy is to close the position immediately after an increase in volatility. The profit and loss profile for different volatilities is illustrated in the following chart:

Profit and Loss Profile on the Last Trading Day, Long Straddle, Option on the Euro Bobl Future June 2002 – P/L in EUR



The dotted-line function depicts the value immediately after the transaction is completed. The profile on the last trading day is already known. The value of the two long positions rises if volatility increases, meaning that a profit will be made (light blue line) regardless of the futures price. The position should be closed as soon as no further short-term increase in volatility is expected. If volatility declines, the profit and loss line will move closer to the profile on the last trading day, as the decrease in volatility and the lapse of time reduce the time value (green line).

Trading Volatility – Maintaining a Delta-Neutral Position with Futures

The value of an option is affected by a number of variables, notably the price of the underlying instrument, the time to expiration and volatility. Knowing this, option traders have devised a selection of different trading strategies, enabling them to trade a view not just on the expected change in the market price of the underlying instrument, but also the development of volatility over time.

If a trader believes that current implied volatility levels (as derived from market prices) are not in keeping with his own forecast, he is able to construct a strategy which will allow him/her to trade the volatility component of an option whilst remaining neutral to market direction over time.

The following example demonstrates how a trader who is bullish of volatility can profit from buying “undervalued” options whilst maintaining a delta-neutral position with the use of futures.

Example

Valuation date	05/03/2002	
Euro Bund Future June 2002	106.20	
Call option on the Euro Bund Future June 2002	106.00	1.32
Delta	0.54	
Implied volatility	8%	

With the use of his option pricing model the trader is able to work out that the current “implied” volatility of the 106.00 Bund call option is eight percent. The trader’s own “forecast” volatility between now and expiration of the option is deemed to be higher. The trader decides to buy the “undervalued” options on the basis that if volatility does increase as expected between now and expiration of the option a profit will ensue.

However, being long of a call means that although the trader is now bullish of volatility, he is also exposed to a fall in the futures price. To eliminate this exposure to the futures price, the trader needs to create a delta-neutral position by which his exposure is purely to volatility. The simplest method for achieving a delta neutral position is to sell an appropriate number of futures contracts (note, long futures have a delta of +1 and short futures have a delta of –1). In line with the option delta of 0.54, the trader needs to sell 54 June futures ($100 \times 0.54 = 54$) to turn the long position of 100 call options into a delta-neutral position.

Options position	Delta	Futures position	Net delta
Buy 100 call options on the Euro Bund Future June 2002 106.00 calls	0.54	Sell 54	0

As time goes by the underlying futures price will rise and fall which means that the delta of the long call will change. Therefore, in order to remain delta-neutral the trader has to regularly rebalance his hedge position. Theoretically the strategy requires continuous adjustment – practically speaking, this would not be feasible due to the trading costs involved. Instead the trader elects to adjust his position depending upon certain tolerance levels (e.g. once a day, or if the position becomes too delta positive or negative, for instance). In the following example we will look at the position over ten trading days, with adjustments taking place once a day.

Volatility Trade					
Day	Futures price	Delta 106 call net delta	Total position delta	Futures adjustment	Futures Profit/loss (ticks)
1	106.20	54	0	Sell 54	-
2	105.68	46	- 8	Buy 8	760
3	106.31	55	+ 9	Sell 9	-288
4	107.00	66	+11	Sell 11	407
5	106.43	56	-10	Buy 10	200
6	105.62	44	-12	Buy 12	1,212
7	105.93	49	+ 5	Sell 5	-350
8	105.31	39	-10	Buy 10	1,320
9	106.00	50	+11	Sell 11	-693
10	106.63	61	+11	Sell 11	-2,322

End of trading period – day ten

Futures price	106.63
106 call premium	1.50

When initiating the strategy, the trader bought 100 Bund 106 call options for a premium of 1.32, with a delta equivalent to 54 futures contracts. To create a delta-neutral position at the end of day one, the trader had to sell 54 futures. On day two the futures price fell to 105.68: this led to a new call delta of 46. This meant that the net delta position at that point was eight contracts short (i.e. $54 - 46$). In order to maintain a delta-neutral position, the trader had to buy eight futures at a price of 105.68. This process of rebalancing was repeated each day for a period of ten days, whereupon the original 100 Bund 106 call options were closed out at premium of 1.50. The futures price on day ten was 106.63. The net result of the whole strategy is summarised below into three categories.

Profit from total futures rebalancing (ticks) = 2,568

The profit / loss on the rebalancing is calculated in ticks. For example, at the end of day two the trader had to buy eight futures at 105.68. The final futures price on day ten was 106.63, therefore the trader made 760 ticks profit from that day's rebalancing: $(106.63 - 105.68) \times 8 \text{ contracts} = 760 \text{ ticks}$.

Loss from original opening futures position (ticks) = (2,322)

The trader originally sold 54 futures at 106.20 to create the delta-neutral position. At the end of day ten the futures were closed out at 106.63, generating a loss of $(106.20 - 106.63) \times 54 = 2,322 \text{ ticks}$.

Profit from options position (ticks) = 1,800

The trader originally bought 100 Bund 106 call options for a premium of 1.32. At the end of day ten the options were closed out at 1.50, creating a profit of $(1.50 - 1.32) \times 100 = 1,800 \text{ ticks}$.

Total profit on the trade (ticks) = 2,046

The total profit on the strategy was 2,046 ticks ($\text{EUR } 10 \times 2,046 = \text{EUR } 20,460$). This was made up from a gain in the original options trade and the net effect of the futures rebalancing, whilst the original futures trade generated a loss.

We can see from the table above that the volatility expressed in the daily futures price over the ten day period was significant and as a result, a profit ensued. It is worth noting that at the outset of a volatility trade such as this, the trader does not actually know precisely where his profit (if any) will come from: the original futures hedge, the option position or rebalancing. The main point is that if volatility does increase over the duration of the trade period, a profit will ensue.

The outlook for a delta-neutral position incorporating a short option position is exactly the opposite: a profit will be made if the actual volatility over the period of the trade is lower than the implied volatility upon the option premium was based. Note that there is no difference regarding the use of calls or puts for this kind of strategy – in practice, a delta-neutral position is often initiated by buying or selling at-the-money straddles.

With the introduction of Delta-Neutral Trading Products within the scope of Eurex Release 6.0, traders will be able to execute delta-neutral options / futures combinations directly in the Eurex trading system.

Hedging Strategies

Options can be used to hedge an exposure right up until the last trading day. Alternatively they can be used on a dynamic basis to hedge an exposure for a shorter duration if required. In addition, options may be used to provide either full or partial protection of a portfolio. The following examples show the flexibility of options hedging.

Hedging Strategies for a Fixed Time Horizon

Rationale

A fund manager has a portfolio of German Government Bonds (Bundesanleihen) worth EUR 40,000,000 under management. Although after a strong price rally, he/she does not rule out further price rises, he/she is looking to hedge his/her profits. Using the sensitivity method, he/she determines a hedge ratio of 435 contracts (cf. chapter on "Determining the Hedge Ratio"). This means that 435 short Euro Bund Futures are required to hedge the position. While this hedge transaction would eliminate the risk exposure, it would also rule out profit potential should prices rise further.

Since the fund manager does not wish to completely neutralize the portfolio, he/she decides to buy put options on the Euro Bund Future. In this way a minimum price is secured for the planned short futures position whose price development most closely matches that of the portfolio to be hedged. At the same time, the profit potential of his/her securities is maintained, albeit reduced by the option premium paid, without any obligation to actually sell them.

If the portfolio value is to be hedged until the option's expiration, the fund manager will apply the ratio of 435 contracts calculated for the futures hedge, as the ratio between the option and futures is 1:1. This approach ignores changes in the option's value during its lifetime.

Initial Situation

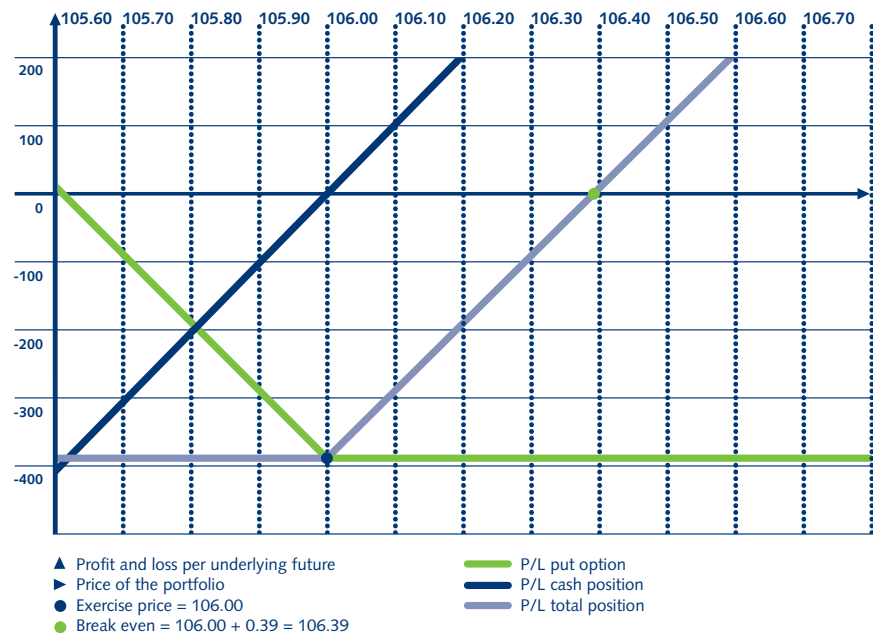
Euro Bund Future June 2002		106.00
Put option on the Euro Bund Future June 2002	106.00	0.39

Strategy

The investor buys put options on the Euro Bund Future June 2002 with an exercise price of 106.00. If the futures price remains unchanged or rises, the result on the overall position is reduced by the paid put premium of 0.39. At the same time, if the futures price falls below the put's exercise price, the loss on the hedged portfolio is limited to this amount. On the assumption that the cash position is matched to the development of the futures contract, the profit and loss profile for the total position – as set out below – is identical to that of a long call on the Euro Bund Future. This is why this combination is also referred to as a “synthetic long call”.

Calculation example: Euro Bund Future June 2002			
Futures price at maturity	Profit/loss on the cash position equivalent to the future	Profit/loss on the 106.00 put option	Total profit/loss on the position
105.20	-0.80	0.41	-0.39
105.30	-0.70	0.31	-0.39
105.40	-0.60	0.21	-0.39
105.50	-0.50	0.11	-0.39
105.60	-0.40	0.01	-0.39
105.70	-0.30	-0.09	-0.39
105.80	-0.20	-0.19	-0.39
105.90	-0.10	-0.29	-0.39
106.00	0	-0.39	-0.39
106.10	0.10	-0.39	-0.29
106.20	0.20	-0.39	-0.19
106.30	0.30	-0.39	-0.09
106.40	0.40	-0.39	0.01
106.50	0.50	-0.39	0.11
106.60	0.60	-0.39	0.21

Profit and Loss Profile on the Last Trading Day, Hedging a Cash Position with Long Put Option on the Euro Bund Future – P/L in EUR



Delta Hedging

If the portfolio value is to be hedged for a certain period of the option's lifetime, changes in value in the cash and option positions must be continuously matched during that period. The delta factor – in other words, the impact of price changes in the underlying instrument on the option price – is particularly important. The delta for an option that is exactly at-the-money is 0.5 (cf. chapter “Delta”). This means that a one unit price change in the underlying instrument leads to a change of 0.5 units in the option price.

On the assumption that, for the sake of simplicity, the cash position behaves in line with a notional hedge position of 435 Euro Bund Futures for a fully hedged cash position, a delta of 0.5 would necessitate the purchase of 2×435 options instead of 435.

As was illustrated in the chapter on “Gamma”, the delta value changes with price change. Hence, the number of options bought has to be adjusted continuously. If, for example, the options move out-of-the-money due to a price rise and the delta falls to 0.25, the option position would have to be increased to 4×435 contracts. This dynamic hedging strategy is referred to as “delta hedging”.

Gamma Hedging

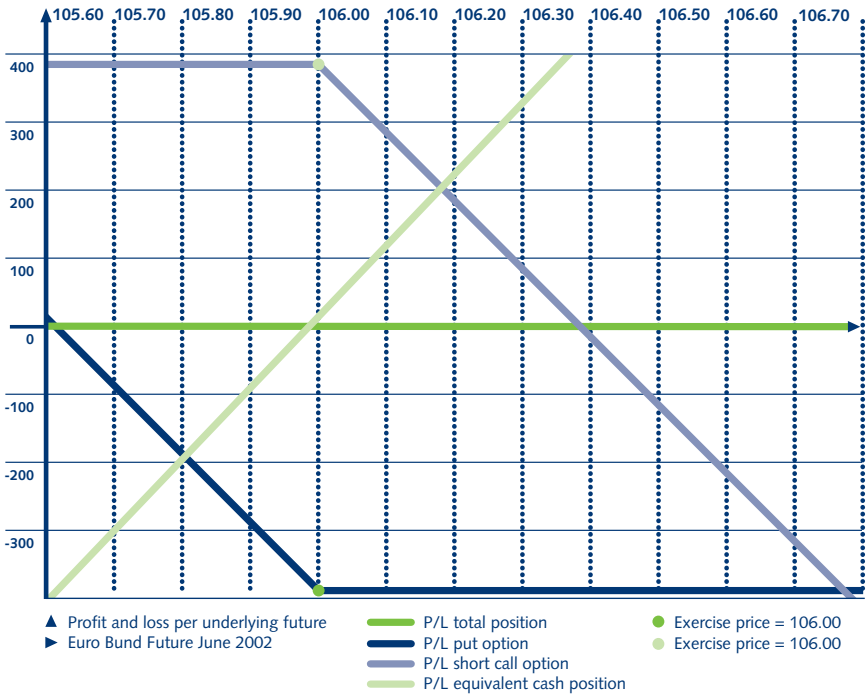
The frequent switching involved in delta hedging results in high transaction costs. The so-called gamma hedge offers the possibility to provide a constant hedge ratio strategy throughout the options' entire lifetime. The purpose of this hedging method is to establish a gamma value for the bond portfolio of zero. The simplest way to achieve this is to hedge a cash position by entering into a long put and a short call position on the corresponding futures contract, with the same exercise price. It is useful to remember that the delta values of both positions always add up to one, which corresponds to a gamma of zero. It is also worth noting that the combination of the long put and the short call is equal to a short futures position. On the basis of the delta hedge example, a call option with an exercise price of 106.00 would be sold additionally, at a price of 0.39. This strategy provides for an offset between the cash position on the one hand and the options position on the other hand. While in the event of falling prices, the cash position suffers a loss that is set off against profits on the options strategy, the opposite is true when prices rise.

Assuming the position is held until expiration, this would result in the following profit and loss pattern:

Calculation example: Euro Bund Future June 2002				
Futures price at maturity	Profit/loss on the cash position equivalent to the future	Profit/loss on the 106.00 put option	Profit/loss on the short 106.00 call option	Profit/loss on the overall position
105.20	-0.80	0.41	0.39	0
105.30	-0.70	0.31	0.39	0
105.40	-0.60	0.21	0.39	0
105.50	-0.50	0.11	0.39	0
105.60	-0.40	0.01	0.39	0
105.70	-0.30	-0.09	0.39	0
105.80	-0.20	-0.19	0.39	0
105.90	-0.10	-0.29	0.39	0
106.00	0	-0.39	0.39	0
106.10	0.10	-0.39	0.29	0
106.20	0.20	-0.39	0.19	0
106.30	0.30	-0.39	0.09	0
106.40	0.40	-0.39	-0.01	0
106.50	0.50	-0.39	-0.11	0
106.60	0.60	-0.39	-0.21	0

The investor makes neither a profit nor loss on this position, irrespective of market development. Because this strategy creates a synthetic short futures contract, the profit/loss profile is equivalent to selling 435 Euro Bund Futures at 106.00. A practical example for this type of position would be a situation where the hedger started out with a Long Put and subsequently wishes to change the characteristics of his position.

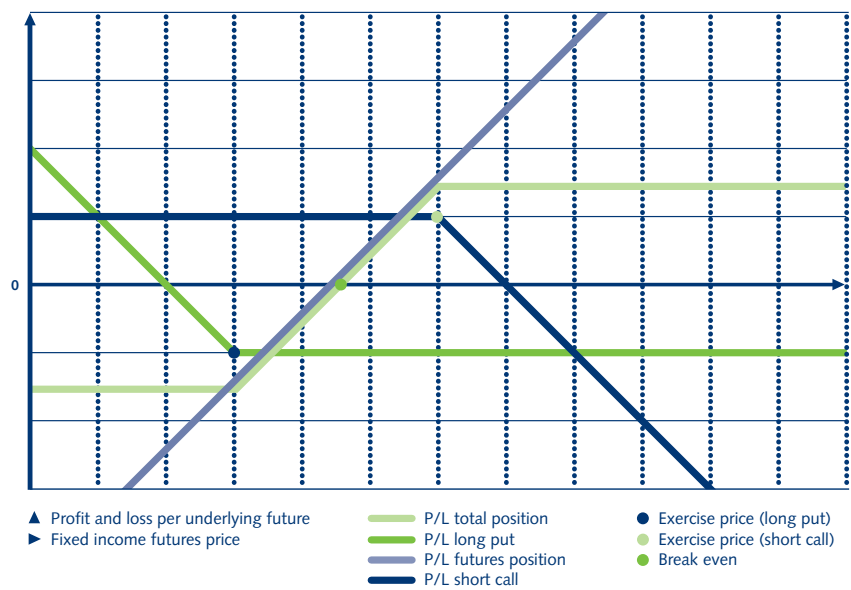
Profit and Loss Profile on the Last Trading Day, Gamma Hedging with the Option on the Euro Bund Future
 June 2002 – P/L in EUR



Zero Cost Collar

Both the delta and gamma hedge strategies provide a full neutralization of the cash position against interest rate or price changes. As an alternative, the portfolio manager can allow his/her position to fluctuate within a tolerance zone, thus only hedging against greater deviations. He/she buys a put with an exercise price below the current market price and sells a call with a higher exercise price for this purpose. A zero cost collar is where the premiums for both options are equal. Options on fixed income futures allow an almost symmetrical interval around the current price in the form of a zero cost collar to be created, provided that transaction costs are ignored.

Profit and Loss Profile on the Last Trading Day, Zero Cost Collar



The profit and loss profile for a long cash position with a collar on expiration is equivalent to a bull spread position.

Futures/Options Relationships, Arbitrage Strategies

Synthetic Fixed Income Options and Futures Positions

Options on fixed income futures give the buyer the right to enter into exactly one contract of the underlying instrument. A call option can be replicated by a put option combined with a future, a put option by a future and a call. A long call and a short put result in a profit and loss profile identical to a long future. Because of the restriction of the option expiration date being prior to the maturity date of the futures contract, such “synthetic” positions can only be held during part of the futures lifetime.

Creating synthetic positions is attractive if mispricing makes them cheaper than the original contract. Price imbalances exceeding transaction costs, thus providing arbitrage opportunities, arise for very short periods of time only and therefore generally available to professional arbitrageurs only. The synthetic positions described in this section mainly serve to illustrate the relationships between options and futures.

Synthetic Long Call

A synthetic long call is created by combining a long futures position with a long put option. Similar to the “real” call, this position is characterized by limited risk exposure and theoretically unlimited profit potential.

Rationale

The investor expects a reduction in 5-year yields. He/she wants to benefit from the expected price increases, at the same time entering into a position with limited risk exposure. This is why he/she decides on a long call position.

Initial Situation

Valuation date	Mid-May 2002	
Euro Bobl Future June 2002	105.10	
Call option on the Euro Bobl Future June 2002	105.00	0.29
Put option on the Euro Bobl Future June 2002	105.00	0.16

Strategy

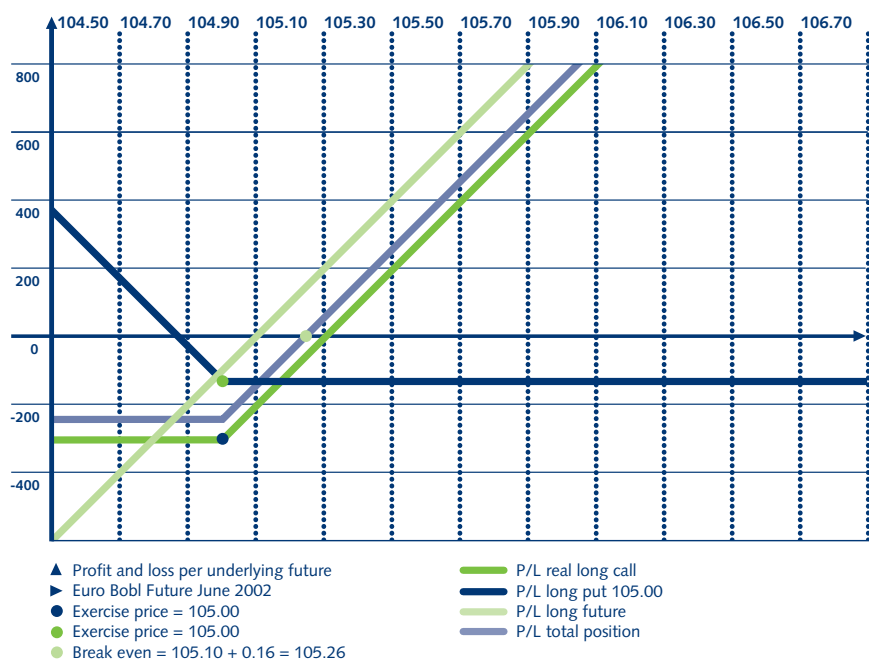
On the basis of prevailing market prices, the investor determines whether a direct call purchase or the synthetic position is more favorable. Both possibilities are displayed in the following table:

Calculation example: Euro Bobl Future June 2002					
Futures price at maturity	Profit/loss on the long future	Value of the 105.00 put	Profit/loss on the 105.00 put	Profit/loss on the synthetic long 105.00 call	Profit/loss on the "real" long 105.00 call
104.50	-0.60	0.50	0.34	-0.26	-0.29
104.60	-0.50	0.40	0.24	-0.26	-0.29
104.70	-0.40	0.30	0.14	-0.26	-0.29
104.80	-0.30	0.20	0.04	-0.26	-0.29
104.90	-0.20	0.10	-0.06	-0.26	-0.29
105.00	-0.10	0	-0.16	-0.26	-0.29
105.10	0	0	-0.16	-0.16	-0.19
105.20	0.10	0	-0.16	-0.06	-0.09
105.30	0.20	0	-0.16	0.04	0.01
105.40	0.30	0	-0.16	0.14	0.11
105.50	0.40	0	-0.16	0.24	0.21

Result

The synthetic long call option has an advantage of 0.03, or EUR 30, over the "real" long call on expiration.

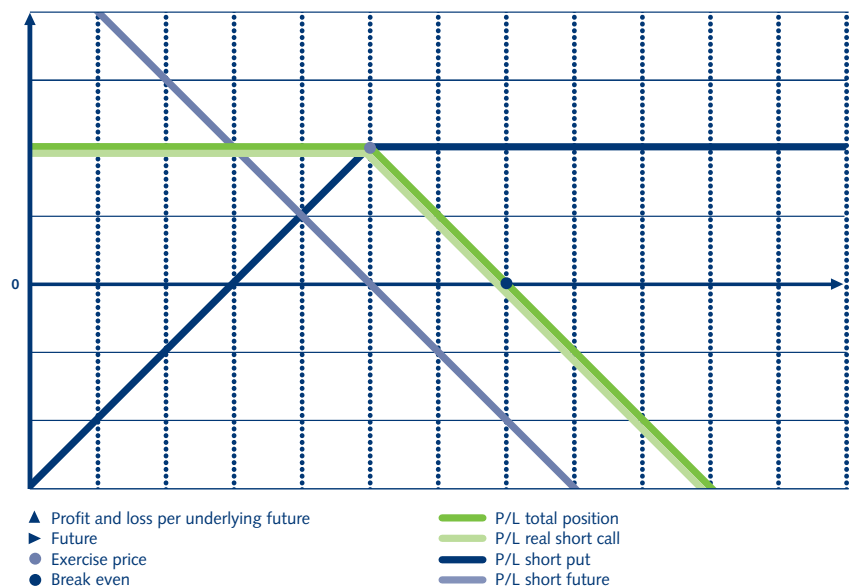
Profit and Loss Profile on the Last Trading Day, Synthetic Long Call, Option on the Euro Bobl Future June 2002 – P/L in EUR



Synthetic Short Call

A synthetic short call is created by combining a short futures position with a short put option. Similar to the “real” short call, the profit potential is limited to the premium received, while the loss potential upon rising prices is unlimited. If he/she expects prices to stagnate or to fall, the investor decides on a short call position, taking the high-risk exposure of this position into account. If a synthetic position can be established more favorably than a direct call purchase, it will be favored by the investor.

Profit and Loss Profile on the Last Trading Day, Synthetic Short Call



Synthetic Long Put

A synthetic long put is created by combining a short futures position with a long call. Similar to all long option positions, the maximum loss is limited to the premium paid. The maximum profit is equivalent to the exercise price less the option premium paid.

Rationale

Mid-May 2002, the investor expects 2-year German capital market yields to rise. He/she wants to benefit from the expected price slump while taking a limited exposure.

Initial Situation

Euro Schatz Future June 2002		102.82
Call option on the Euro Schatz Future June 2002	102.75	0.14
Put option on the Euro Schatz Future June 2002	102.75	0.10

Strategy

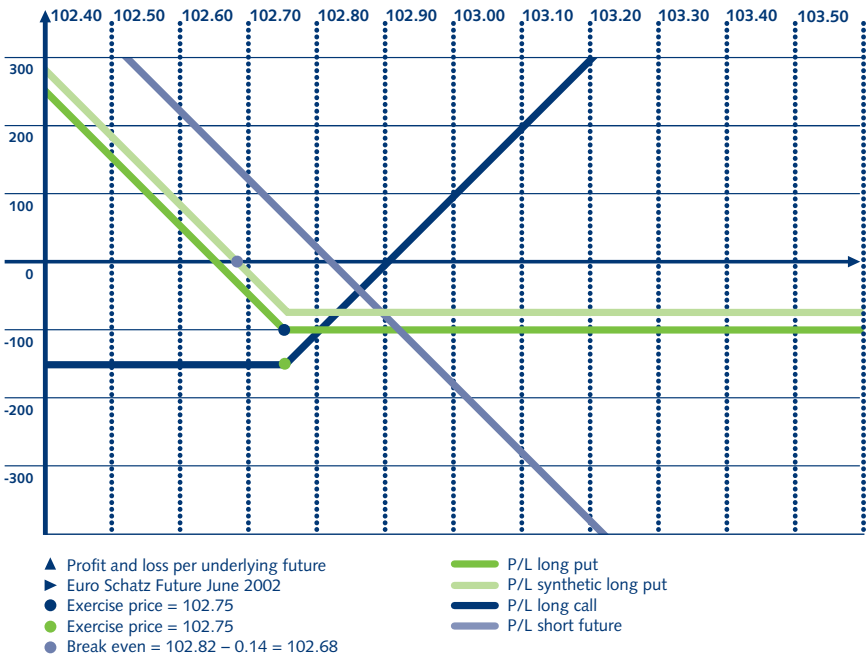
The investor decides to buy a put option on the Euro Schatz Future, comparing the "real" position with the synthetic put:

Calculation Example: Euro Schatz Future June 2002					
Futures price at maturity	Profit/loss on the short future	Value of the 102.75 call	Profit/loss on the 102.75 call	Profit/loss on the synthetic long 102.75 put	Profit/loss on the "real" long 102.75 put
102.40	0.42	0	-0.14	0.28	0.25
102.50	0.32	0	-0.14	0.18	0.15
102.60	0.22	0	-0.14	0.08	0.05
102.70	0.12	0	-0.14	-0.02	-0.05
102.80	0.02	0.05	-0.09	-0.07	-0.10
102.90	-0.08	0.15	0.01	-0.07	-0.10
103.00	-0.18	0.25	0.11	-0.07	-0.10
103.10	-0.28	0.35	0.21	-0.07	-0.10
103.20	-0.38	0.45	0.31	-0.07	-0.10
103.30	-0.48	0.55	0.41	-0.07	-0.10
103.40	-0.58	0.65	0.51	-0.07	-0.10

Result

The synthetic put has an advantage of 0.03, or EUR 30 per contract, over the "real" put. For this reason, the investor decides on the synthetic long put position. This is only a model calculation. In practice, high transaction costs can make creating a synthetic position less favorable.

Profit and Loss Profile on the Last Trading Day, Synthetic Long Put, Option on the Euro Schatz Future
 June 2002 – P/L in EUR

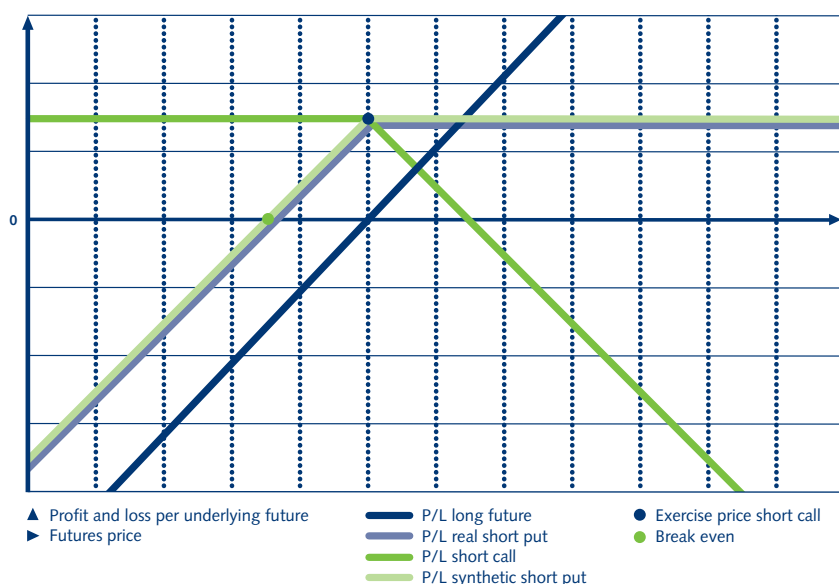


Synthetic Short Put

A synthetic short put is a combination of a long futures position and a short call option. The maximum loss incurred at expiry is equal to the exercise price less the premium received. The profit potential is limited to the premium received.

Comparing the real and synthetic positions is in line with the examples illustrated above.

Profit and Loss Profile on the Last Trading Day, Synthetic Short Put



Synthetic Long Future/Reversal

Synthetic futures positions are created by combining a long and short option position. As a rule, the bid/offer spread for options is wider than for futures contracts. This is why synthetic futures positions are hardly used as trading strategies, but almost exclusively for arbitrage purposes to exploit any mispricing of options.

A long futures position can be reproduced by combining options positions which match the characteristics of the long future position – a long call option provides profit participation on the upside, while a short put position comprises risk exposure in the event of falling prices.

Rationale

Having analyzed the price structure for options on the Euro Bund Future, an arbitrageur identifies the June 2002 106.50 put option as overpriced in relation to the corresponding call. As a result, the synthetic futures contract is cheaper than the actual Euro Bund Future.

Initial Situation

Call option on the Euro Bund Future June 2002	106.50	0.26
Put option on the Euro Bund Future June 2002	106.50	0.52
Euro Bund Future June 2002		106.29

Strategy

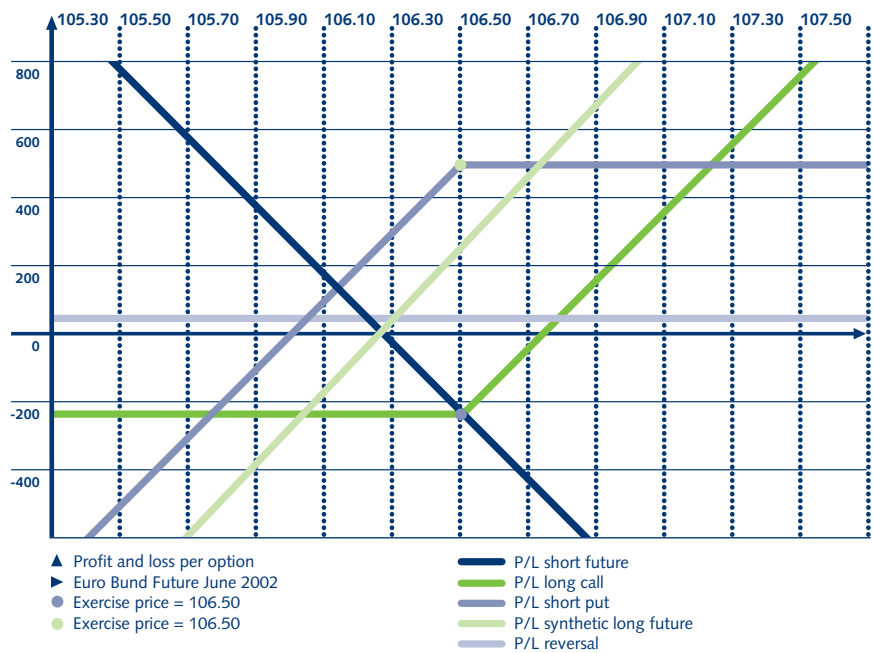
The arbitrageur buys the synthetic futures contract and simultaneously sells the “real” futures contract. This arbitrage strategy is called a “reversal”.

Calculation example: Euro Bund Future June 2002					
Futures price at maturity	Profit/loss on the “real” short future	Profit/loss on the long 106.50 call	Profit/loss on the short 106.50 put	Profit/loss on the synthetic long future	Profit/loss on the reversal strategy
105.60	0.69	-0.26	-0.38	-0.64	0.05
105.70	0.59	-0.26	-0.28	-0.54	0.05
105.80	0.49	-0.26	-0.18	-0.44	0.05
105.90	0.39	-0.26	-0.08	-0.34	0.05
106.00	0.29	-0.26	0.02	-0.24	0.05
106.10	0.19	-0.26	0.12	-0.14	0.05
106.20	0.09	-0.26	0.22	-0.04	0.05
106.30	-0.01	-0.26	0.32	0.06	0.05
106.40	-0.11	-0.26	0.42	0.16	0.05
106.50	-0.21	-0.26	0.52	0.26	0.05
106.60	-0.31	-0.16	0.52	0.36	0.05
106.70	-0.41	-0.06	0.52	0.46	0.05
106.80	-0.51	0.04	0.52	0.56	0.05
106.90	-0.61	0.14	0.52	0.66	0.05

Result

Regardless of the price development of the Euro Bund Future, a profit of 0.05 is made on each arbitrage unit (consisting of one contract each of long call, short put and short futures). Note that this calculation does not take into account bid/offer spreads or transaction costs.

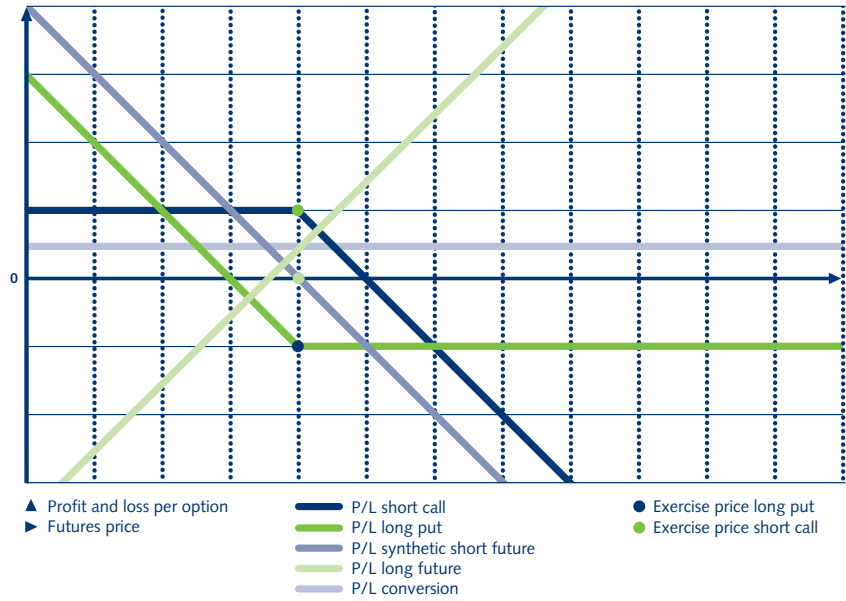
Profit and Loss Profile on the Last Trading Day, Reversal, Option on the Euro Bund Future June 2002 – P/L in EUR



Synthetic Short Future/Conversion

A synthetic short futures position is created by combining a short call with a long put option. Where a call option is expensive or a put option is cheap (both in relative terms), a profit can be achieved by creating a synthetic short futures position and combining it with a “real” long futures contract. This strategy, which is called a “conversion”, is the opposite of a “reversal” strategy.

Profit and Loss Profile on the Last Trading Day, Conversion



Synthetic Options and Futures Positions – Overview

The components of the above mentioned synthetic positions are summarized in the following table:

Synthetic created by call option	put option	future
long call	-	long	long
short call	-	short	short
long put	long	-	short
short put	short	-	long
long future	long	short	-
short future	short	long	-

The table clearly illustrates that “mirror” positions, for example long call and short call, are created by opposing component positions.

Glossary

Accrued interest

The interest accrued from the last interest payment date to the valuation date.

Additional margin

Additional margin is designed to cover the additional potential closeout costs which might be incurred. Such potential closeout costs would arise if, based on the current market value of the portfolio, the expected least favorable price development (worst case loss) were to materialize within 24 hours. Additional margin is applicable for options on futures (options settled using “futures-style” premium posting) and non-spread futures positions.

American-style option

An option which can be exercised on any exchange trading day before expiration.

At-the-money

An option whose exercise price is identical to the price of the underlying instrument.

Basis

The difference between the price of the underlying instrument and the corresponding futures price. In the case of fixed income futures, the futures price must be multiplied by the conversion factor.

Bond

Borrowing on the capital market which is certificated in the form of securities vesting creditors' claims.

Call option

In the case of options on fixed income futures, this is a contract that gives the buyer the right to enter into a long position in the underlying futures contract at a set price on, or up to a given date.

Cash-and-carry arbitrage

Creating a risk-free or neutral position by exploiting mispricing on the cash or derivatives market, by simultaneously buying bonds and selling the corresponding futures contract.

Cash settlement

Settling a contract whereby a cash sum is paid or received instead of physically delivering the underlying instrument. In the case of a financial futures contract (for example, the Euribor Future), cash settlement is determined on the basis of the final settlement price.

Cheapest-to-deliver (CTD)

The deliverable bond for which delivery is most attractive in terms of cost.

Clean price

Present value of a bond, less accrued interest.

Closeout

Liquidating (closing) a short or long option or futures position by entering into a counter position.

Conversion factor (price factor)

The price factor multiplied by 100 is the theoretical price, assuming a yield of 6 percent, of the bond on the delivery date of the corresponding contract. Using this factor for conversion purposes allows the different deliverable bonds to be comparable with respect to their different coupons and remaining lifetimes in the event of physical delivery.

Convexity

Parameter used to take the non-linear price-yield correlation into account when calculating the interest rate sensitivity of fixed income securities.

Cost of carry

The difference between financing costs and the income received on the cash position (net financing costs).

Coupon

(i) Nominal interest rate of a bond. (ii) Part of the bond certificate vesting the right to receive interest.

Cross Hedge

Strategy where the hedge position does not precisely offset the performance of the hedged portfolio due to the stipulation of integer numbers of contracts or the incongruity of cash securities and futures and/or options.

Daily settlement price

The daily valuation price of futures and options, as determined by Eurex, on which the determination of daily margin requirements is based.

Delta

The change in the option price in the event of a one point change in the underlying instrument.

Derivative

Financial instrument whose value is based on one underlying instrument from which they are derived. Hence the expression derivatives.

Discounting

Calculating the present value of the future cash flows of a financial instrument.

European-style option

An option which can only be exercised on the last trading day.

Exercise

The option holder's declaration to either buy or sell the underlying instruments at the conditions set in the option contract.

Exercise price (strike price)

The price at which the underlying instrument is received or delivered when an option is exercised.

Expiration date

The date on which the option right expires.

Final settlement price

The price on the last trading day, which is determined by Eurex according to specified guidelines.

(Financial) futures

A standardized contract for the delivery or receipt of a specific amount of a financial instrument at a set price on a certain date in the future.

Futures spread margin

This margin must be pledged to cover the maximum expected loss within 24 hours, which could be incurred on a futures spread position.

Futures-style premium posting

The (remaining) option premium is not paid until exercise or expiration. This method is used by Eurex Clearing AG for options on futures.

Greeks

Option risk parameters (sensitivity measures) expressed by Greek letters.

Hedge ratio

The number of futures contracts required to hedge a cash position.

Hedging

Using a strategy to protect an existing portfolio or planned investments against unfavorable price changes.

Historical volatility

Standard deviation of returns of an underlying instrument (based on empirical data).

Implied volatility

The extent of the forecast price changes of an underlying instrument which is implied by (and can be calculated on the basis of) current option prices.

Inter-product spread

=> Spread Positions.

In-the-money

An option whose intrinsic value is greater than zero.

Intrinsic value

The intrinsic value of an option is equal to the difference between the current price of the underlying instrument and the option's exercise price. The intrinsic value is always greater than or equal to zero.

Leverage effect

The leverage effect allows participants on derivatives markets to enter into a much larger underlying instrument position using a comparably small investment. The impact of the leverage effect is that the percentage change in the profits and losses on options and futures is greater than the corresponding change in the underlying instrument.

Lifetime

The period of time from the bond issue until the redemption of the nominal value.

Long position

An open buyer's position in a forward contract.

Macaulay duration

An indicator used to calculate the interest rate sensitivity of fixed income securities, assuming a flat yield curve and a linear price/yield correlation.

Margin

Collateral, which must be pledged as cover for contract fulfillment (Additional Margin, Futures Spread Margin), or daily settlement of profits and losses (Variation Margin).

Mark-to-market

The daily revaluation of futures positions after the close of trading to calculate the daily profits and losses on those positions.

Maturity date

The date on which the obligations defined in the futures contract are due (delivery/cash settlement).

Maturity range

Classification of deliverable bonds according to their remaining lifetime.

Modified duration

A measure of the interest rate sensitivity of a bond, quoted in percent. It records the change in the bond price on the basis of changes in market yields.

Option

The right to buy (call) or to sell (put) a specific number of units of a specific underlying instrument at a fixed price on, or up to a specified date.

Option price

The price (premium) paid for the right to buy or sell.

Out-of-the-money

A call option where the price of the underlying instrument is lower than the exercise price. In the case of a put option, the price of the underlying instrument is higher than the exercise price.

Premium

=> Option price.

Present value

The value of a security, as determined by its aggregate discounted repayments.

Put option

An option contract, giving the holder the right to sell a fixed number of units of the underlying instrument at a set price on or up to a set date (physical delivery).

Remaining lifetime

The remaining period of time until redemption of bonds which have already been issued.

Reverse cash-and-carry arbitrage

Creating a neutral position by exploiting mispricing on the cash or derivatives market, by simultaneously selling bonds and buying the corresponding futures contract (opposite of => Cash-and-carry arbitrage).

Risk based margining

Calculation method to determine collateral to cover the risks taken.

Short position

An open seller's position in a forward contract.

Spread positions

In the case of options, the simultaneous purchase and sale of option contracts with different exercise prices and/or different expirations.

In the case of a financial futures contract, the simultaneous purchase and sale of futures with the same underlying instrument but with different maturity dates (time spread) or of different futures (inter-product spread).

Straddle

The purchase or sale of an equal number of calls and puts on the same underlying instrument with the same exercise price and expiration.

Strangle

The purchase or sale of an equal number of calls and puts on the same underlying instrument with the same expiration, but with different exercise prices.

Synthetic position

Using other derivative contracts to reproduce an option or futures position.

Time spread

=> Spread positions.

Time value

The component of the option price arising from the possibility that the investor's expectations will be fulfilled during the remaining lifetime. The longer the remaining lifetime, the higher the option price. This is due to the remaining time during which the value of the underlying instrument can rise or fall (a possible exception exists for options on futures and deep-in-the-money puts).

Underlying instrument

The financial instrument on which an option or futures contracts is based.

Variation margin

The profit or loss arising from the daily revaluation of futures or options on futures (mark-to-market).

Volatility

The extent of the actual or forecast price fluctuation of a financial instrument (underlying instrument). The volatility of a financial instrument can vary, depending on the period of time on which it is based. Either the historical or implied volatility can be calculated.

Worst-case loss

The expected maximum closeout loss that might be incurred until the next exchange trading day (covered by additional margin).

Yield curve

The graphic description of the relationship between the remaining lifetime and yields of bonds.

Appendix 1: Valuation Formulae and Indicators

Single-Period Remaining Lifetime

$$P_t = \frac{N + c_1}{(1 + {}^t r_{c1})}$$

- P_t Present value of the bond
- N Nominal value
- c_1 Coupon
- ${}^t r_{c1}$ Yield from the time period t_0 until t_1

Multi-Period Remaining Lifetime

$$P_t = \frac{c_1}{(1 + {}^t r_{c1})^{t1}} + \frac{c_2}{(1 + {}^t r_{c2})^{t2}} + \dots + \frac{N + c_n}{(1 + {}^t r_{cn})^{tn}}$$

- P_t Present value of the bond
- N Nominal value
- c_n Coupon at time n
- ${}^t r_{cn}$ Average yield from the time period t_0 until t_n

Macaulay Duration

$$\text{Macaulay-Duration} = \frac{\frac{c_1}{(1 + {}^t r_{c1})^{t_{c1}}} \times t_{c1} + \frac{c_2}{(1 + {}^t r_{c1})^{t_{c2}}} \times t_{c2} + \dots + \frac{c_n + N}{(1 + {}^t r_{c1})^{t_{cn}}} \times t_{cn}}{P_t}$$

- P_t Present value of the bond
- N Nominal value
- c_n Coupon at time n
- ${}^t r_{cn}$ Average yield from the time period t_0 until t_n
- t_n Remaining lifetime of coupon c_n

Convexity

$$\text{Convexity} = \frac{\frac{c_1}{(1 + {}^t r_{c1})^{t_{c1}}} \times t_{c1} \times (t_{c1} + 1) + \frac{c_2}{(1 + {}^t r_{c1})^{t_{c2}}} \times t_{c2} \times (t_{c2} + 1) + \dots + \frac{c_n + N}{(1 + {}^t r_{c1})^{t_{cn}}} \times t_{cn} \times (t_{cn} + 1)}{P_t (1 + {}^t r_{c1})^2}$$

- P_t Present value of the bond
- N Nominal value
- c_n Coupon at time n
- ${}^t r_{cn}$ Average yield from the time period t_0 until t_n
- t_n Payment date of coupon c_n

Appendix 2: Conversion Factors

Bonds Denominated in Euros

$$\text{Conversion factor} = \frac{1}{(1.06)^f} \times \left[\frac{c}{100} \times \frac{\delta_i}{\text{act}_2} + \frac{c}{6} \times \left(1.06 - \frac{1}{(1.06)^n} \right) + \frac{1}{(1.06)^n} \right] - \frac{c}{100} \times \left(\frac{\delta_i}{\text{act}_2} - \frac{\delta_e}{\text{act}_1} \right)$$

Definition:

δ_e	NCD1y-DD
act_1	NCD – NCD1y, where $\delta_e < 0$ NCD1y – NCD2y, where $\delta_e \geq 0$
δ_i	NCD1y – LCD
act_2	NCD – NCD1y, where $\delta_i < 0$ NCD1y – NCD2y, where $\delta_i \geq 0$
f	$1 + \delta_e / \text{act}_1$
c	Coupon
n	Integer years from the NCD until the maturity date of the bond
DD	Delivery date
NCD	Next coupon date
NCD1y	1 year before the NCD
NCD2y	2 years before the NCD
LCD	Last coupon date before the delivery date

Bonds Denominated in Swiss Francs

$$\text{Conversion factor} = \frac{1}{(1.06)^f} \times \left[\frac{c}{6} \times \left(1.06 - \frac{1}{(1.06)^n} \right) + \frac{1}{(1.06)^n} \right] + \frac{c(1-f)}{100}$$

Definition:

n	Number of integer years until maturity of the bond
f	Number of full months until the next coupon date, divided by 12 (except for $f = 0$, where $f = 1$ and $n = n - 1$)
c	Coupon

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Brochures and Learning Tools

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Brochure

Eurex – The European Derivatives Market

Eurex – Products

Eurex Quick Reference Guide Trading

Eurex Quick Reference Guide Clearing

Interest Rate Derivatives –

Fixed Income Trading Strategies

Equity and Equity Index Derivatives –

Trading Strategies

Eurex – Training Brochure

Eurex – Trading Calendar

Location

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Entrance Hall > Publications > Brochures

Market Place > Products > Trading Calendar

Applications

Eurex MarginCalculator

Location

Market Place > Clearing and Settlement
> Risk Based Margining

The following educational tools can be ordered on CD Rom via the Learning Portal:

All about Eurex Options

All about Eurex Futures

Eurex StrategyMaster

Eurex MarginCalculator

Eurex OptionAlligator (option price calculator)

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