

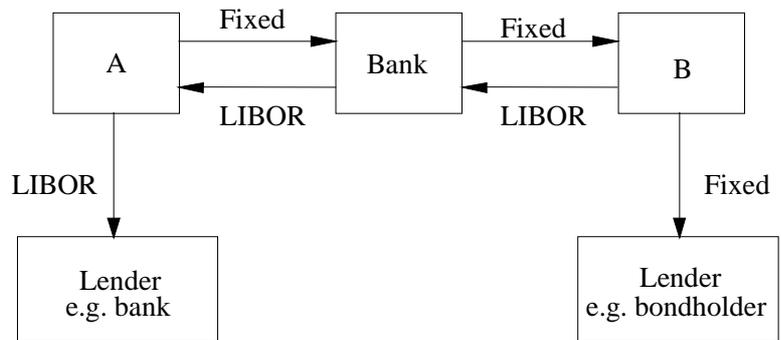


**Module VII:
SWAPS**

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INTEREST RATE SWAPS

An interest rate swap is an agreement between two parties to exchange a fixed payment for a floating payment.



Example

Company A agrees to pay Company B 8% a year for 5 years on \$10 million in return for B paying A 6-month LIBOR on the same sum.

The interest rate swap arose in the early 1980s. Banks were happy lending to some borrowers who were unable to raise fixed rate funds through the bond market owing to investor aversion. However, banks were happier lending floating funds. It made sense, therefore for a company who wanted to borrow fixed funds, but who could not conveniently do so, to find another company who wanted to borrow floating funds. The latter company would issue a fixed rate bond and the former would issue a floating rate note (or else borrow direct from its bankers), and the two companies would agree to swap.

Example

Company A wants fixed funds.
Company B wants floating funds.

A would borrow fixed at 8% and floating at 6-month LIBOR + ½%.

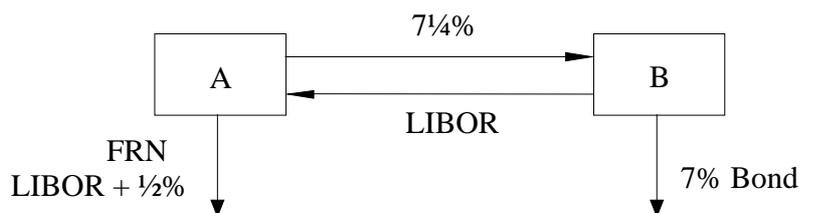
B would borrow fixed at 7% and floating at 6-month LIBOR flat.

A issues an FRN at LIBOR + ½%. B issues a fixed rate bond at 7%. A agrees to pay B 7.25% and B agrees to pay A 6-month LIBOR flat.

A's net cost of funds: $+(L + 0.5) - L + 7.25 = 7.75\%$ (saving 0.25%)

B's net cost of funds: $+7\% - 7.25\% + L = \text{LIBOR} - 0.25\%$ (saving 0.25%)

The swap rate is thus 7.25% annual versus 6-month LIBOR.



Because there is this differential between relative cost of funds in different markets, which is a form of arbitrage, both companies can save money against their normal cost of funds. Note also that the total saving available, 0.50%, is equal to the difference between the fixed rate differential and the floating rate differential. This savings calculation is useful as it allows an instant determination of the amount available to be shared between the two companies.

If A borrowed at 8% and LIBOR plus 0.75% and B borrowed at 7.75% and LIBOR plus 0.25%, the total saving available is

$$(0.75\% - 0.25\%) - (8\% - 7.75\%) = 0.25\%$$

if B issues an FRN and A issues a bond and they then swap.

In the first example, they both saved 0.25%. The savings do not, of course, need to be apportioned equally.

Clearly it is inefficient for the two companies to be chasing around looking for each other. What used to happen is that each would go to a brokerage or securities house and the house would try to match counterparties, receiving a fee for arranging the transaction. The fee would either be a single up-front fee or, more commonly, would be taken every year out of the flow of payments between the two clients.

Nowadays the swap markets are somewhat more liquid than this. Instead of waiting for a counterparty, a trading house simply quotes a swap rate to its clients and the clients can decide themselves whether to trade.

The swap rate itself “emerges” from normal market operations.

Quotations and conventions

Swaps are quoted in two ways. The first is as an outright rate, e.g. 7.75 - 7.80. The trading house would receive fixed at 7.80% or pay fixed at 7.75% versus paying/receiving LIBOR. The second is as a spread over a reference rate such as the US Treasury yield for the period of the swap. So one might see a 5-year swap rate as "45-50". If the US Treasury 5-year yield is 7.00%, then the effective swap rate is 7.45 - 7.50 *semi-annual*. If an annual swap rate is wanted, this rate must be converted to an annual yield.

The savings across different bases are not the same: 0.25% saved on an annual cost of funds is not the same as 0.25% saved on a semi-annual cost, and in addition, for those markets where the money market day count differs from bond day counts, 0.25% saved against either annual or semi-annual bond rates is not the same as 0.25% saved against LIBOR.

A number of adjustments have to be made before one can compare the savings in different markets.

The simplest way to look at an interest rate swap is as a combination of a fixed rate bond asset/liability and an FRN liability/asset. In terms of interest rate risk, the receiver of fixed is in the same position as a trader who is long a fixed rate bond with a coupon and maturity equal to that of the swap and who has funded his position, not overnight but instead for a period of time equal to the time to the next floating payment date.

The effects of payment conventions on swap spreads

A saving of 20 b.p. versus a LIBOR-based cost of funds is not the same as a saving of 20 b.p. versus an annual fixed rate cost of funds for two reasons. The first is that, depending on market convention, the day count for deposits is not the same as the day count for bonds. The second is that account must be taken of periodicity: 20 b.p. versus 6-month LIBOR is not the same as 20 b.p. on an annual basis. How, therefore, can the basis points be equated ?

Consider a company that normally accesses the bond markets at 8% annual and through a swap is shown a saving of 25 b.p. in dollar LIBOR terms. Therefore, somebody is paying him 25 b.p. on a money market basis, equivalent to $(365/360 \times 25)$, 25.35 b.p. Now this 25.35 b.p. cannot be converted to an annual spread/saving without considering the absolute level of rates.

Suppose the "base" rate is 8% semi-annual. Therefore the saving generates a rate of 7.7465%.

8% annualised: 8.160%

7.7465% annualised: 7.900%

Annual spread: $8.160\% - 7.900\% = 0.260\% = 26 \text{ b.p.}$

So the saving on the swap generates a net cost of funds of $(8 - 0.26)$, 7.74%.

Lest you think that this 1 b.p. makes little difference, consider that for a 10-year bond at a yield level of 8%, 1 b.p. amounts to roughly 7 cents, easily enough to make the difference between a trade and no trade.

If instead the general level of rates is 16%...

16% annualised: 16.640%

15.7465% annualised: 16.366%

Annual spread: $16.640\% - 16.366\% = 27.4 \text{ b.p.}$

Clearly the saving is dependent on underlying yield levels, although it is not particularly sensitive to them: a change in yield from 8% to 16% produces a change of only about 2½ b.p.

Consider instead a bank which normally accesses the money market and through a swap is shown a saving in annual fixed rates of 25 b.p. Clearly, the conversion works in reverse, and again some “base” rate needs to be assumed.

8% semi-annualised: 7.846%

7.75% semi-annualised: 7.605%

Semi-annual spread: $7.846\% - 7.605\% = 24.1$ b.p.

Money market spread: $24.1 \times 360 \div 365 = 23.8$ b.p.

So from the perspective of the bank, the swap has produced a saving of 23.8 b.p. versus its usual LIBOR-based cost of funds.

(When selecting the base rate, it is reasonable to choose some suitable nearby integer. Little extra advantage is gleaned by using an actual fixed rate.)

SWAP-DRIVEN ISSUANCE**Corporate bond market effects of swap-driven issuance**

As a consequence of the interest rate swap market, corporate bond spreads nowadays correlate fairly strongly with swap spreads.

Suppose that AA corporates can issue fixed at T+50 b.p., floating at LIBOR + 10 b.p., and swap spreads are at T+40 b.p., so that there is no advantage in issuing in one market rather than the other. Now suppose that owing to the absence of corporate fixed rate issuance, or to increased demand for fixed rate debt corporate spreads narrow to T+25 b.p. Then corporate issuers who would normally issue at LIBOR+10 floating will instead issue fixed at T+25 and receive at T+40 versus LIBOR, generating a cost of funds of LIBOR-15 b.p.

As a consequence, with demand to receive fixed from these issuers accompanying increased fixed rate issuance, corporate spreads will widen again and swap spreads will come in, until equilibrium is again reached, say at T+35 for corporates and T+25 for swaps. So ultimately both spreads come in by 15 b.p.

Or else, suppose corporate spreads widen out to T+80 b.p. Then potential fixed rate issuers will now issue FRNs at L+10 b.p. and pay T+40 b.p. on a swap, generating a cost of funds of T+50 b.p., a saving of 30 b.p. on ordinary debt issuance. Consequently, fixed rate issuance will dry up and with more issuers paying fixed, bond spreads will come in and swap spreads will widen out until equilibrium is reached, which may be at, say bonds at T+70 b.p. and swaps at T+60. So both will have widened by 20 b.p.

Hence in both cases swap spreads and corporate spreads must be linked. It should be observed that owing to the relative insensitivity of floating rate investors it is unlikely that floating spreads will move much during the corporate/fixed spread movements.

ANALYSIS AND VALUATION OF SWAPS

Valuing positions and the duration of a swap

When valuing positions, all the fixed cash flows on the swaps can be offset as far as possible, and the balance of fixed flows are valued on the basis of the difference between the original rates and the appropriate current swap rates. This produces a series of flows which can be valued in the usual way (using the zero curve for example). A positive value means that the swap trader is making money on his positions, a negative value means that he is losing money.

A simpler approach is the summation of the mark-to-market values of the individual swaps. But how do you work this out ?

Example

A two-year \$10 million swap pays fixed at 8% per annum. Rates rise to 9%. How much has the swap declined in value ?

The swap receiver has a loss of 1% per annum on \$10,000,000, which is \$100,000 cash per annum. This is PV'd at the prevailing swap rate (or the zero curve, &c.).

$$\$100,000 \div 1.09 + \$100,000 \div 1.09^2 = \$175,911 = 1.76\%$$

We can now derive the duration formula for swaps from first principles.

By definition,

PV of $d\%$ in years 1, 2, 3... = $d\%$ move x mod. duration

If $D = 1 + \text{swap rate} \div 100$

$$\therefore d\% \div D + d\% \div D^2 + d\% \div D^3 \dots + d\% \div D^{\text{life}} = d\% \times \text{MD}$$

$$\therefore \text{MD} = (1/D + 1/D^2 + 1/D^3 \dots + 1/D^{\text{life}}) = (1 - 1/D^{\text{life}}) \times 100 \div \text{swap rate}$$

So in the example above, for a swap originally priced at 8%

$$(1 - 1/1.08^2) \times 100 \div 8 = 1.78 \text{ years.}$$

In fact, this formula for modified duration may be used to work out the duration of *any* bond trading at par on a coupon date, if one replaces swap rate with bond yield..

For a semi-annual swap, the calculation is slightly modified:

$$MD = (1 - 1/(1 + SR\%/200)^{2 \times \text{life}}) \times 100 \div SR\%.$$

so for an 8% 2-year semi-annual swap, the calculation is

$$(1 - 1/1.04^4) \times 100 \div 8 = 1.81$$

Following on from this connection between swap duration and bond duration, the performance of swaps can be used to demonstrate other properties of bonds. Notice that the actual amount of the loss on the swap was less than that “predicted” by the duration. This is (obviously) due to convexity. But in the context of the swap, convexity arises owing to the nature of the valuation process.

From the point of view of the receiver, if rates rise, although he loses so much money p.a., the loss is PV'd to the present day at a high rate of interest, whereas if rates fall to produce the same amount of future p.a. profit as was lost in the previous case, these profits are PV'd at a lower rate to produce a higher PV than the PV when discounting at a higher rate.

So far as dollar duration is concerned, the price of a swap in bond-equivalent terms is simply 100 plus or minus the percentage mark-to-market value. A payer of fixed on a profit of 4% is in effect short a bond at par currently trading at 96. A receiver of fixed on a profit of 3% is in effect long a bond at par currently trading at 103.

Using the zero curve

In the example above, the cash flows were discounted at a constant rate of 9%. This discounting was done on the basis of a single swap rate. If you had viewed the cash flows as a stream of zeroes, then you would have valued each piece of flow using the yields of the appropriate zeroes: cash is cash and should be valued as such.

Allowing for the duration of the floating leg (not)

The floating leg has, of course, a (Macaulay) duration equal to the time to next refix. In much of the literature the net duration of a swap is commonly quoted as the combination of the two durations, so if the fixed leg has a duration of 7 years (to a receiver) and the floating, 0.5 years, then the duration overall would be stated to be 6.5 years.

While this mathematically correct, it is in practical terms stupid. Simply, a 7-year duration interest rate and a 6-month rate do not have a high enough correlation for one to be able to combine the sensitivities to both as a single number. It is far better to say that a swap has a pair of durations, short-end and long-end, and hedge them (if so desired) separately. (Capital regulations that use durations for determining limits, e.g. the EC CAD treat swaps for purposes of interest rate risk as if they were two products, a long dated fixed asset/liability and a short-term liability/asset.)

TRADING AND HEDGING USING SWAPS

Trading

There is no reason why a swap cannot be traded as a bond other than derivatophobia.

A swap is little different from a financed bond position, which will make or lose money in accordance with an interest rate view. The main difference is that bond traders tend to finance their position short term (overnight or 1 week, for example) whereas the swap position includes an implicit term financing of 3 or 6 months depending on the LIBOR tenor.

So there is a yield curve risk on a totally unhedged position. If a trader uses the fixed cash flows to pay LIBOR, when LIBOR rises with no change in the swap rate, the swap costs cash, even though no mark-to-market loss would initially hit the trading account. In substance this is little different to the position of a trader who, hoping to benefit from positive carry, finds that his funding cost has risen but his bonds have not moved in price.

There are some advantages to using swaps rather than bonds for investors as well as traders. It is easier to pay fixed on a swap than it is to short a bond. (Furthermore, some investors are permitted to enter into a swap who are not allowed to short bonds.) Swaps can also be used to leverage the position.

Hedging

There are a number of good reasons to use swaps for hedging purposes. Shorting bonds may not always be possible. In general both traders and investors like to use only a few hedging securities to hedge a large number of bonds: a trader with a position in 50 or 60 different securities will often want to hedge the net position with just one bond/swap per duration bucket.

For the size of the hedge, swaps are far more liquid than any non-Treasury bond.

Given that there is a higher correlation between general eurobond yields and swap rates than there is between eurobond yields and government yields, the spread risk component of the hedge (risk) is somewhat reduced if you use swaps to hedge eurobonds or domestic swaps to hedge domestic corporate bonds rather than Treasury issues.

Example

3-year 7% UST note yields 7.00% at its present price of 100.
3-year swap spreads (semi-annual): +50 b.p.

Two investors each own \$10,000,000 nominal of a particular 3-year corporate bond with a coupon of 7.50%, currently trading at par (and thus yielding 7.5% semi-annual).

Investor A shorts \$10,000,000 UST.
Investor B pays fixed on a \$10,000,000 swap at a rate of (7.00 + 0.50), 7.5%.

UST yields rise 100 b.p., swap spreads widen by 25 b.p., and corporate spreads widen by 50 b.p.

3-year 10% UST note yields 8.00% = price of 97.4
3-year swap spreads : + 75 b.p.
3-year corporate bond yields 9.00% = price of 96.13

Investor A: P&L

corporate	: \$10,000,000 x (96.13 - 100)	= -\$387,000
Treasury	: \$10,000,000 x (100 - 97.4)	= \$260,000
Net P&L	: \$260,000 - \$387,000	= -\$127,000

Investor B: P&L

corporate	: \$10,000,000 x (96.13 - 100)	= -\$387,000
<i>swap rate: 8.00 + 0.75 = 8.75%; price of 7.50% 3-year bond at 8.75% semi.: 96.76 (using 3-year rate for convenience)</i>		
swap	: \$10,000,000 x (100 - 96.76)	= \$324,000
Net P&L	: \$324,000 - \$387,000	= -\$63,000

As corporate spreads widened, both investors lost money. However, investor B lost less because the swap spread widened as well.

The following table shows different instruments and their appropriateness (or otherwise) for hedging. It is not necessarily universally applicable.

Hedging instrument	Pro	Con
Treasury	liquid; negligible transaction costs; efficient repo market	Spread risk
Corporate bond	lowest spread risk	bid/offer spread illiquid in large hedging amounts; inefficient repo market
Bond futures	most liquid negligible transaction cost; transparency	largest spread/ basis risk
Interest rate swap	low cost; low spread risk; liquid in large size	bid/offer spread high vs. Tsy. investor/trader perception

CURRENCY SWAPS

The standard currency swap consists of an agreement to pay a fixed amount of one currency per annum against receiving a fixed amount of another currency per annum followed by an exchange of principal on maturity of the swap. It is commonplace, but not essential, to exchange currency on opening the swap.

The currency swap originated in the 1970s as a way around exchange controls. Multi-national companies could exploit currency swaps in effect to transfer funds borrowed or available from one country to another.

In effect all currency flows are paid at an exchange rate agreed in advance. If the agreement is for a 5-year currency swap of £100mm versus \$160mm and paying of £8mm versus receiving \$12mm, this is in effect the same as a £100mm swap paying 8% p.a. in sterling against receiving (\$12mm ÷ \$160mm), 7.5% p.a. in dollars, the whole swap at an exchange rate of GBP/USD1.6000.

The excess of coupon received over coupon paid in the different currencies is offset, theoretically at least by the supposed appreciation of the lower paying currency over the lifetime of the swap. As an example, consider the swap above. If there is a ½% difference between 5-year rates, then you could expect that the 5-year currency forward would be about GBP/USD1.56.

The receiver of the higher sterling amount (£8mm versus paying \$12mm, i.e. initially £7.5) finds that after 5 years he has to pay out \$160mm of a dollar which should then be worth that forward rate, i.e. around £102.5mm and he will only receive £100mm. If he hedges now the final amount (and indeed all the cash flows), he will not be buying dollars at 1.6000 but at that forward rate of 1.56. And indeed, one incentive to enter into a currency swap is one the basis of a view on the currency: if the dollar payer believes that the dollar will stay around 1.6000 (or weakens), he will make money.

Basis points

The basis point saving in one currency is not the same as in another where the interest rate in the two currencies is different: this may be appreciated by considering that if one PVs the savings to produce a direct comparison, the PV will depend on the rate of discount used.

Example

A swap structure gives an issuer the choice of a 20 b.p. p.a. saving in dollars or in yen for 10 years. Long term dollar rates are 8% and yen rates, 5%.

PV of 20 b.p. pa. for 10 years discounted at:

5%: 154 b.p.

8%: 134 b.p.

So in order to compare the savings across currencies, it is necessary to find a saving in one currency that produces the same PV as the basis point equivalent in another currency. This can be done by PVing the saving in one currency at the appropriate rate and then re-annuitising it at the other rate to produce the equivalent saving in the second currency.

Example

What is the equivalent basis point saving in yen of a 20 b.p. p.a. saving in dollars when dollar rates are 8% and yen rates 5% ?

PV in dollars: 134 b.p.

p.a. value of an annuity worth 134 b.p. today at a rate of 5%:
17.35 b.p.

So a swap showing a saving of 20 b.p. in dollars saves only 17.35 b.p. in yen.

Quotation/translation

Consider a simple currency swap dollars versus sterling. Now consider a simple interest rate swap fixed versus floating dollars. If you chain the two swaps, you can actually generate a fixed sterling rate versus dollar LIBOR.

All swaps can be chained like this, so it is quite possible, indeed normal, to refer to *all* swap rates against 6-month dollar LIBOR.

CREDIT IMPLICATIONS OF SWAPS

There are two essential components to credit risk: the likelihood of the credit event, and the amount lost in that event. Where the former has a high probability but the latter a low amount, the expected loss may be somewhat less than where the risk is low but the loss in that event is high.

In the case of a swap, what matters to a counterparty in the event of a default is the *netted out* accrued interest due to the default date, and the replacement cost of the swap: what it costs to put on a new swap for the same tenor.

Example

A firm is receiving 8% annual fixed vs. 6-month LIBOR for what was a 10-year swap but is now a 9-year swap. LIBOR for the preceding 6 months (182 days) was 4% p.a. If the counterparty defaults, the firm was due to pay $(4\% \times 182/360)$, 2.02% and receive 8%, so the accrued interest potentially lost is $(8 - 2.02)$, 5.98% of the notional swap amount.

Suppose that, first, 9-year swap rates are now at 7%. The firm receiving would now (in theory) have to replace the existing 8% swap with the new 7% swap. Assuming 7% is the yield to value all the fixed flows, the loss of 1% a year for 9 years is worth 6.52%.

Hence an additional loss of 6.52% above the 5.98% (total 12.5%) would be taken. This may be a real loss, if there was a countervailing trade, or an economic loss, if the swap was an outright trade.

Suppose, however, that 9-year swap rates had risen to 10%. Then a new 9-year swap would generate a *profit*, real or economic, of 2% a year, worth 11.52%, and so the net outcome on the swap default would be $(11.52 - 5.98)$, +6.54%. (In practice, therefore, a liquidator would attempt to keep running the swap as an asset of the defaulting company, but this may not be possible if there is already a delinquency on the coupon payment.)

The point of this example is that first, the amount lost is a small proportion of the notional amount, and second, circumstances may arise where a default is actually favourable.

This is similar to counterparty risk for a securities transaction. If one sells a bond DVP to a client, the counterparty risk is linked not only to his likelihood of default, but also to the possible change of bond price between trade and settlement. If one sells a bond and two days later the client disappears, by which time the bond has fallen in price by 5%, one only loses that 5%, not the full bond value. And if the bond rises by 5%, one makes money (legal considerations apart....)

If instead the firm in the example above had bought a bond and financed it instead, then a default on the bond may cost the entire principal. In addition the firm has a risk to itself - financing risk, for where one's own credit quality declines, that will not affect the swap payments, whereas one's ability to refinance may be sufficiently affected that one may have to liquidate the financed asset when it may not be propitious to do so.

That interest rate swaps' exposure is somewhat less than the notional amount does *not* mean that the same is true for currency swaps.. Owing to the exchange of principal on termination, it is possible for the exposure/replacement cost to exceed the original local currency notional amount. Suppose that a currency swap is for \$100mm vs. EU100mm, and over ten years the euro appreciates to \$4. Then the net value of the exchange on termination is \$300mm, somewhat greater than the original local currency \$100mm.

This explains why counterparty limits are of far greater significance in currency swaps than interest rate swaps, and why some firms will trade with (almost) anyone in the latter, while they will require a AAA-rated counterparty for the former.

Currency swaps - additional risks

You can appreciate that the risks on a currency swap are greater than on an interest rate swap. On a fixed-fixed currency swap you have two fixed rates and a cross currency risk to worry about, rather than a single fixed rate. In addition, as currency risk is much higher than interest rate risk, the associated credit risk on a currency swap is higher.

As with interest rate swaps, what is at risk is the replacement cost of the swap: how much it would cost you to put on the same swap with another counterparty in the event of a default.

The replacement cost is unlikely to be high with the interest rate swap. However, as it is possible for one currency to depreciate significantly against another, 50% not being unknown over even short time scales, the replacement cost of a currency swap can be a substantial proportion of the original nominal value. As a rule of thumb, the replacement risk on a currency swap is an order of magnitude greater than on an interest rate swap.

This increased risk is allowed for modern capital regulations. As a result, the capital commitment required for a long term currency swap, or rather, the cost of capital deployed, will have a major impact on the economics of the swap: so much so that on many occasions when a currency swap appears to be the perfect theoretical solution to a funding or A/LM problem, the capital required by the securities firm to trade makes the practical swap uneconomic.

SYNTHESIS

There appears to be no obvious connection between yield curve analysis and swap rates, other than the fact that one can presumably plot swap rates against life to create a swap curve, or generate a zero swap curve from the par curve.

However, the techniques divulged in the yield curve analysis module can be used to generate swap rates: hence, the heading "synthesis".

Suppose that 3-month LIBOR is 5% and the eurodollar futures contract expiring in 3 months is currently at 94.5. This futures price implies a LIBOR rate of $(100 - 94.5) \times 5.5\%$ 3 months forward.

Now given that we have a rate of 5% for the first 3 months, and 5.5% for the second 3 months, we can work out the value of \$1 in 6 months invested and re-invested:

$$\text{\$1} \times (1 + 5/100 \times 91/360)(1 + 5.5/100 \times 91/360) = \text{\$1.0267}$$

$$\text{(Annualised return: } (\text{\$1.0267} - \text{\$1}) \times 36000/182 = 5.28\%)$$

Suppose also that the eurodollar contract expiring in 6 months is at 94, i.e., 6%, and the contract expiring in 9 months is at 93.8, i.e., 6.2%.

Investing \$1 over the entire year:

$$\begin{aligned} &\text{\$1} \times \\ &(1 + 5/100 \times 91/360) \times \\ &(1 + 5.5/100 \times 91/360) \times \\ &(1 + 6/100 \times 91/360) \times \\ &(1 + 6.2/100 \times 92/360) \\ &= \text{\$1.0588} \\ &\text{i.e., an annual yield of 5.88\%} \end{aligned}$$

In other words, from that series of 3-month LIBOR rates one can generate annual zero coupon rates. Clearly once we have enough 3-m LIBOR rates we can generate zero coupon rates as far out as we like, and we can then, from the zero rates, convert to par rates using the technique covered earlier.

Now in the days when futures were not liquid beyond about 2 years, and swap rates were driven purely by internal supply and demand, the above calculations were of questionable use.

Nowadays, however, as futures are liquid and swap rates link to bond and deposit markets, such calculations are integral: in fact, on occasion changes in swap rates will drive far forward futures prices, for if there is no change, risk-free arbitrage becomes possible.

To show how this might work, suppose that the 1-year swap rate vs. 3-month LIBOR were to move from 5.88% to say 5.25%. Someone could simultaneously pay fixed at 5.5%, thus receiving 3-month LIBOR, and would buy a strip of futures at 94.5, 94, and 93.8.

In 3 month's time the spot 3-m LIBOR of 5% would be received, a cash value of about **+1.268%** allowing for day counts. Suppose that at this time 3-m LIBOR were now 6%. He therefore loses 0.5 on the futures p.a., i.e., **-0.125%** cash.

In 6 months he will receive that 6%, a cash amount of **+1.52%**. Suppose, 3-m LIBOR is now 4%. The futures will close at 96, so he will earn 2% p.a., i.e., **+0.5%** cash.

In 9 months, he receives the 4% LIBOR fixed 3 months previously, i.e., **+1.014%**. Suppose spot 3-m LIBOR is now 5%. The last futures contract will close at 95, so he receives 1.2% p.a., i.e., **+0.3%** cash.

Finally, in 12 months, he pays **-5.25%** and receives 5% p.a. for 3-months, **+1.268%**

Net P&L ignoring PV and re-investment effects:
 $+1.268 - 0.125 + 1.52 + 0.5 + 1.014 + 0.3 - 5.25 + 1.268 = +0.50$.

This profit is earned risk-free: hence at a swap rate of 5.25% arbitrageurs will be buying futures, which will move the price up.

ASSET SWAPS

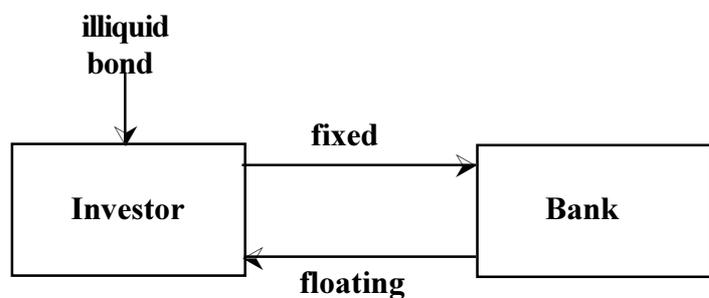
Investors frequently have illiquid bonds in their portfolios. If they think that yields will rise, and they therefore would like to sell the bonds, by definition these bonds will not be sold at prices which reflect their “true” value, the value excluding illiquidity premium.

Accordingly, an investors may decide to hedge his holding in such bonds by, for example, paying fixed on a swap. In his mind, he has converted a bond so hedged into a floating rate asset.

An asset swap serves to make explicit the mental process undergone: the investor agrees to pass on bond coupons in exchange for receiving LIBOR. An adjustment must of course be made to take account of the actual value of the bond.

Suppose a bond yields 175 b.p. over USTs when swap spreads are UST plus 75 b.p. Therefore the bond is trading at about 100 b.p. over LIBOR (allowing for periodicity, day counts, &c.). If the bond is issued by a borrower who could be expected to issue/ trade at LIBOR plus 50 b.p., for example, the asset swap does not look too bad (although it may well crystallise a loss on the existing holding).

Classic pattern



Such assets need not be existing holdings. It occasionally happens that swappable assets may be available in the bond markets for traders and investors to buy. Historically, most of the swapped bonds were low coupon bonds issued with equity warrants. Once the warrants had been sold away from the bonds-plus-warrants, package, the bonds traded at substantial discounts to par. These bonds were mostly issued by Japanese corporate borrowers, and had guarantees from Japanese banks. In the days when these banks were of very acceptable credit quality, this made asset swapping very attractive.

In the last few years, falling yields in bond markets resulted in many bonds trading substantially above par. Many investors are reluctant to invest in bonds trading at too high a premium, and so the yields on such issues rose relative to the rest of the market. Accordingly the bulk of asset swap activity in 1992 and 1993 was in high coupon issues. The simplest asset swap is a par swap, where the cash value of the asset equals the nominal value of the swap. As you can deduce from the above comments, the majority of assets available for being swapped are trading away from par, and the asset swap has to be structured to allow for this. If the bond is trading below par, the owner of the asset usually has to pay a lump sum to the swap counterparty to compensate for the lower coupons paid, but he will benefit from the asset's appreciation to par. If the bond is trading above par, the swap counterparty pays a lump sum, in return for which he receives high fixed coupons.

Asset swaps in practice

For a number of reasons, not least of which is the transparency of operation for the client, the majority of asset swaps at Morgan Stanley are par swaps, even though the asset underlying the swap is not itself trading at par.

For example, if the bond were trading at 90, had a 6% coupon, and could be swapped at LIBOR + 50 b.p., a securities firm, e.g., Morgan Stanley would sell the bond to the client at 100, receive 6% coupons over the life of the swap/asset and pay the client LIBOR + 50 b.p. Now under such circumstances the fixed rate on a swap is likely to be somewhat higher than 6%. In other words

MorganStanley is receiving less than it should on the fixed side. (And clearly, the NPV of such a swap would not be 0.)

In the literature, it is usually assumed that such a non-par swap would therefore require the client to pay a lump sum to Morgan Stanley at the beginning (or if required, at the end) of the swap in compensation. In practice, this is unnecessary: this notional lump sum has in fact been paid simply by the investor's buying the bond at par, rather than at market. (See the section below on market and par pricing.)

The question as to the proper margin over LIBOR that an asset is worth was originally, though wrongly, answered by comparing the yield on the asset with the current swap rate, the difference being the swap margin. So if the yield on the bond had been 8% when the swap rate had been 7.75%, it would have been assumed that the asset was 25 over LIBOR (allowing for day counts, &c.).

Such a calculation is misleading similarly to the way that the direct yield comparison of any two bonds with equal maturities but widely different coupons is misleading. The swap rate is a par rate: to value the bond away from the par, one must use the zero curve, and discount the flows at the zero curve yields. The difference between the PV so generated and the actual price of the bond (allowing for accrued interest) represents the PV of the future margin payments.

Example

A 5-year bond has a 6% coupon and is trading at 90. The zero curve extracted from the swap curve is as follows:

1 year : 7.00%
2 years : 7.25%
3 years : 7.50%
4 years : 7.75%
5 years : 8.00%

$$6 \times (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4) + 106/1.08^5 = 92.25$$

So the PV of the future margins is 2.25. This is re-annuitised using the zero curve:

$$2.25 \div (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4 + 1/1.08^5) = 0.558$$

So the margin over LIBOR would be 0.558, or 55.8 b.p. (annual bond basis).

By contrast, using the pure yield method, the implied 5-year swap rate is 7.923 and the YTM on the bond is 8.540, a spread of 61.7.

An alternative approach to the calculation is to observe that the firm offering the swap has to pay the fixed rate of 7.92 to the market on a countervailing swap, pays a margin over LIBOR to the asset swap buyer, receives a coupon of 6% p.a. on the asset swap and receives 10% (100% - 90%) as upfront compensation. Hence:

$$10\% = (7.92\% - 6\% + m) \times (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4 + 1/1.08^5)$$

And as before $m = 55.8$ b.p.

Market and par pricing

On occasion, particularly when the bond is far away from par, then the asset will be sold to the client at market: some investors in particular are unable to buy or sell securities at off-market prices, regardless of rationale. On termination of the swap/maturity of the asset, the asset will be returned to Morgan Stanley at that market price.

For example if the asset were a low coupon asset trading at 70, the client would buy the asset at 70, pay the coupons over to Morgan Stanley, receive the LIBOR-plus payments, and on maturity the asset would revert to Morgan Stanley at 70. That instant appreciation to 100 that Morgan Stanley would receive would compensate it for the presumed net negative cash flow for the duration of the swap resulting from paying a current LIBOR(-plus) versus receiving a low coupon.

The drawback to this is that Morgan Stanley is taking a risk on the asset, which in general we do not wish to do following the initial sale, so market price swaps for sub-par bonds are uncommon.

For bonds above par, the risk is not the same, as Morgan Stanley is taking a capital loss anyway, and if the investor disappears on maturity, the consequences are far from adverse.

Suppose that for technical reasons the asset swap is sold at 90. Now the firm is making its 10% lump sum only at the end of the swap, so there must be a recalculation. But in addition the investor is not paying a par amount, so will not be paid an asset swap rate on par, but on the value of his investment. Therefore the pricing has to be radically adjusted.

The most logical approach is to treat the bond as though 90 were par. Therefore the coupon is $(6/90\%)$, 6.67%. And if the asset swap is for \$5mm nominal, the countervailing swap would be for $(\$5\text{mm} \times 90\%)$, \$4.5mm. Finally, on maturity, the investor hands over the 10% appreciation, worth 11.11% of the new nominal amount.

Now the securities firm loses (7.92 - 6.67%) 1.25% plus margin a year, and receives a FV of 11.11%.

$$(1.25\% + m) \times (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4 + 1/1.08^5) = 11.11/1.08^5$$

So m is 61.9 b.p.

The assumption here is that the securities firm has a financing cost of LIBOR, of course.

Suppose instead we were dealing with a high coupon bond.

Example

A 14% 5-year bond at 120:

For a par priced swap, the securities firm loses 20 points, but receives (14% - 7.92), 6.08% less margin.

$$(6.08 - m) \times (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4 + 1/1.08^5) = 20\%$$

m = 111.6 b.p.

For a market price swap on \$5mm nominal, countervailing swap is for (\$5mm x 120), \$6mm, and the coupon is (14%/120%), 11.67%.

Firm receives (11.67% - 7.92%), 4.75% less margin, loses (20%/120), 16.67% in 5 years.

$$(4.75 - m) \times (1/1.07 + 1/1.0725^2 + 1/1.075^3 + 1/1.0775^4 + 1/1.08^5) = 16.67/1.08^5$$

m = 92.9 b.p.

Now it may appear as if the market price swap gives a lower spread for the higher coupon bond, and the higher spread for the lower coupon bond than the par swap. Though in a sense it is true, the appearance is distorted by the difference in nominal amounts.

In the case of the low coupon bond, on the par swap the 55.8 b.p. was earned on a par value of \$5mm, equal to \$27,900 a year. For the market price swap, the 61.9 b.p. was earned on an invested value of \$4.5mm, equal to...\$27,900 a year.

And for the high coupon bond, the par swap earned 111.6 b.p. on \$5mm, \$55,800, whereas the market price swap was 92.9 b.p. on \$6mm, also \$55,800.

Asset swapping non-standard instruments

Floating rate instruments

Although some asset swaps are in effect reverse swaps, where the owner of the floating rate asset swaps to fixed, it is far more common to encounter asset swaps where one floating index is swapped for another, through the medium of a basis swap.

There is no particular difficulty in the structuring, although not all basis swaps are necessarily that liquid. The general problem with pricing basis swaps is that the usual arbitrage-driven pricing principles do not apply. There is no natural defined relationship between T-bills and LIBOR rates other than the latter's almost always being higher than the former (and exceptions not being worth worrying about.) Once a rate is arrived at, however, then the structuring of the asset swap is itself straightforward.

Taxable bonds

In some markets swappable assets have taxable coupons. Where the tax cannot be reclaimed, then for yield/cash flow purposes the post-tax coupon must be used.

In some tax regimes, although tax is withheld, it can be reclaimed. In this event, the delay in reclamation must be factored into the cash flow analysis. For example, Buoni del Tesori Poliennali (BTPs), Italian government fixed rate bonds, used to have 12.5% withholding tax. The average delay in reclamation was 45 days. If 45-day money is 10%, and the pre-tax coupon were 8%, tax withheld would be $(8\% \times 12.5\%)$, 1%, and this would be worth $(1\% \div (1 + 10\% \times 45/360))$, .988%. today. However, this calculation operated on the assumption that the reclamation is actually successful.

Where there is a risk that the reclamation is unsuccessful, it must be agreed at the start of the swap whether the client (the payer of the fixed coupon) must gross up the coupon himself, or, much more commonly, the receiver (Morgan Stanley) will assume the risk itself, and will then be entitled to receive any tax reclaimed.

In return for assuming that risk, Morgan Stanley will value the tax component of the coupon at some discount from its actual value. Following on from the example above, the coupon for analytic purposes (spread calculations, &c.) would be the post-tax coupon of 7% plus some discounted value of the 1% tax. If the “normal” discount were 65%, then when determining the yield/spread on the asset, the assumed coupon would be $(7\% + 1\% \times 65\%)$, 7.65%, but Morgan Stanley would be entitled to the full amount of the tax reclaimed. If the reclamation is successful, then Morgan Stanley would profit by 0.35%.

(Suppose that the spread on the 8% asset were LIBOR plus 150 b.p., the post-tax spread assuming the taxable component were valued at 0.65% would be roughly LIBOR plus 85 b.p. The client would pay the 7% post-tax coupon to Morgan Stanley, who would pay the client LIBOR plus 85 b.p., and would then attempt to claim back the full value. If Morgan Stanley could not, for whatever reason, claim the tax back, it would in effect have lost 65 b.p. If it received the full amount, clearly that would represent a profit of 35 b.p.)

Taxable FRNs

Where a floating asset is swapped into fixed, or into another floating base, the impact of withholding tax may be significant when rates rise.

Example

Certificati di Credito del Tesoro (CCTs) are Italian government FRNs fixed over the Italian T-bill (Buoni Ordinari del Tesoro or BOTs) yields. (When these were subject to 12.5% withholding tax they were thus fixed over BOT gross yields.) If the pre-tax coupon were 8%, then the tax would have been, as before, 1%. If, however, the pre-tax coupon were 16% as a result of sharply higher yields, the tax would be 2%.

Consequently one cannot simply allow a fixed margin for the taxable coupon as with the fixed asset in the earlier example. A possible solution (though not a standard one) is to incorporate some form of cap written by the holder (payer) of the taxable instrument, so

that when rates rise sufficiently for the tax effect to be significant, the payer will be paying out on the cap as well.

Callable bonds

The presence of a call on what is post-swap a LIBOR-plus FRN deterrent to a floating rate receiver, so the fixed rate receiver can comfortably incorporate a call (i.e., in the event of the assets being called, the swap terminates). The firm offering the swap must themselves buy a call, and factor into the LIBOR spread the cost of that call. The call strike will reflect the level at which the call on the asset is likely to be exercised. Presumably the asset will be called when the YTM from the call price/date to maturity is higher than the refinancing rate.

Example

A particular corporate borrower normally is expected to borrow at LIBOR plus 50 b.p. It issued a 10% bond a while ago that now has 5 years to run but is callable at 102 in 3 years.

YTM of 2-year 10% bond at 102: 8.87%

Therefore when swap rates go below (8.87% - 0.50%), 8.37%, the bond will (should) be called. The firm, therefore, in addition to paying fixed on a 5-year swap to the market to offset the asset swap, would buy a swaption to receive 8.37% for 2 years in 3 years' time.

(Strictly speaking, the pricing should be determined by forward zero curves, but the effect of greater accuracy is marginal.)

The cost of the option reduces the PV available to be re-annuitised, and therefore (obviously) the spread on the swap.

FORWARD SWAPS

If a treasurer wanted to lock in now the rate at which he would borrow money market funds in a few months' time, he might use a FRA or a forward-forward. Where the treasurer wants to lock in now the rate at which he would borrow money long term he could attempt to trade the bond equivalent of a forward-forward. The problem is that no firm is likely to commit to a bond issue say 6 months or 2 years forward, for reasons of capital adequacy as much as anything else.

A good solution to the treasurer's problem is thus to hedge against long term rates through, e.g., futures or a swap which comes into effect at the time when the company wishes to borrow, i.e., a forward swap. The only difficulty is how such a swap should be priced.

In fact, the calculation is identified to that for forward par curves you may have encountered in earlier sessions

The current swap rates are used as the par curve, the spot zero curve is generated, then the forward zero curve, and finally the forward par curve. It should be observed that the rate so hedged is not the rate that the company may actually have locked in as a borrowing cost, but is instead the reference rate with respect to which the company may borrow. (This is covered in depth in the section on accreting swaps.)

Swaptions

A swaption is simply an option on a swap. As the forward price of the underlying is used to price an ordinary option, so the forward swap rate is used to price a swaption.

Banks tend to make money as interest rates rise, owing to their deposit bases tending to provide cheap funding. (Personal deposit rates also tend to be put up more slowly by banks than their interest charges, and much more slowly than commercial rates.)

For the corporate treasurer with a large amount of floating

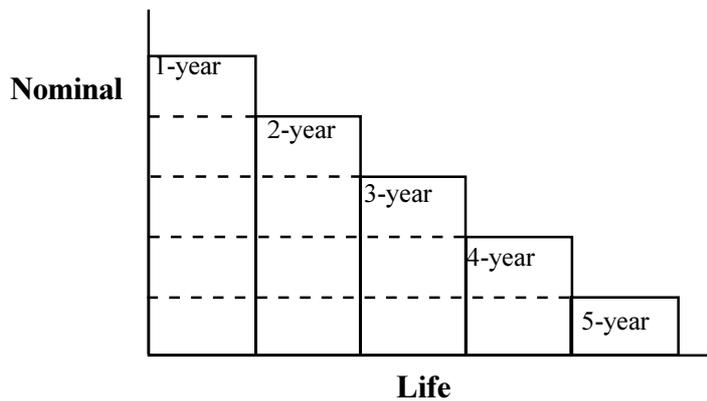
rate borrowings, the purchase of a swaption provides immunity against future interest rate increases.

For the swapped issuer, the ability to buy a swaption allows him to offer an investor a put provision on his debt without the risk of his bond issue becoming “unswapped” by the exercise of the put: for if the investors exercise the put, the borrower would exercise his swaption, effectively reversing the swap he entered into at the time of issue.

AMORTISING SWAPS

An amortising swap is simply a swap with a decreasing principal amount.

Amortising swaps tend to be priced from the arithmetic mean of the swap rates for the different periods, weighted by maturity.



The construction is relatively straightforward.

A typical use of an amortising swap may be to asset-swap an amortising bond or loan.

Example

An investor owns \$20,000,000 nominal of an 8% bond which redeems 10% of its principal in year 1, 20% in year 2, 30% in year 3 and the last 40% in year 4. The bond is currently trading at par. Current swap rates are as follows:

- 1 year : 6.00%
- 2 years : 6.50%
- 3 years : 7.00%
- 4 years : 7.50%

As the bond is illiquid, he decides to put on a swap converting the cash flows from fixed to floating.

Accordingly, the investor puts on a 4-year amortising swap for \$20,000,000.

Period years	nominal value of swap \$,000,000		swap rate %.	principal weighting \$,000,000	
	amortising plain			by life	by life and rate
A	B		C	A x B	A x B x C
1	20	2	6.00	2	12
2	18	4	6.50	8	52
3	14	6	7.00	18	126
4	8	8	7.50	32	240
Total				60	430

Average swap rate: $430 \div 60 = 7.17\%$

Thus if he pays 7.17% versus LIBOR, he will receive a net LIBOR plus $(8.00 - 7.17)$, 0.83% (ignoring day-count effects).

The drawback of this calculation is that it is a pure arithmetical calculation with insufficient allowance being made for the longer period having the higher rate.

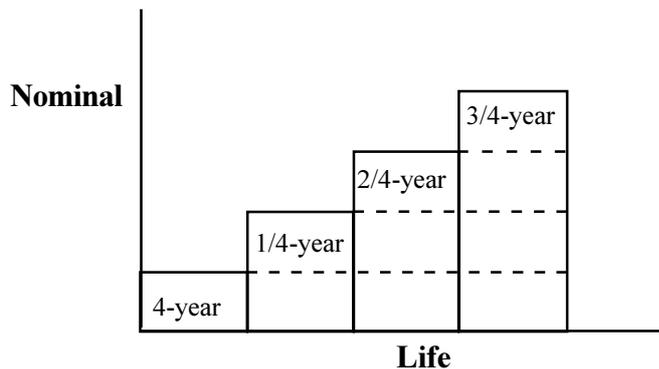
More accurately, one should calculate the IRR of the theoretical bonds underlying the swap, i.e., \$2mm of 6% 1-year bond, \$4mm of a 6.5% 2-year bond, \$6mm of a 7% 3-year bond, and \$8mm of a 7.5% 4-year bond. This gives an IRR of....7.16%

Cash flows \$,000

	1-year	2-year	3-year	4-year	Total
Year 0	-\$2,000	-\$4,000	-\$6,000	-\$8,000	-\$20,000
Year 1	\$2,120	\$260	\$420	\$600	\$3,400
Year 2		\$4,260	\$420	\$600	\$5,280
Year 3			\$6,420	\$600	\$7,020
Year 4				\$8,600	\$8,600

ACCRETING SWAPS

An accreting swap is an amortising swap in reverse: the principal amount increases every year. For example, a swap starting off for \$5mm accretes every year for 3 years at \$5mm a year. Thus the initial swap is \$5mm for 4 years, after 1 year it is now a \$10mm swap for 3 years, after 2 years a \$15mm swap for 2 years, and after 3 years a \$20mm swap for 1 year.



The swap rates used to determine the average rate are not, unlike on the amortising swap, the spot rates, but are instead the appropriate forward rates. The above accreting swap is clearly a \$5mm 4 year swap, a \$5mm 3-year swap 1 year forward, a \$5mm 2-year swap 2 years forward and a \$5mm 1-year swap 3 years forward.

Once those rates are generated, then the calculation of the average rate on the accretion may be carried out. As with the amortising swap, a pure arithmetical cash weighted average is used.

Example

Swap rates:

- 4 years : 8.00%
- 1/4 year : 8.40%
- 2/4 year : 8.50%
- 3/4 year : 8.80%

Period years	nominal value of swap \$,000,000		swap rate %.	principal weighting \$,000,000	
	amortising plain			by life	by life and rate
A	B		C	A x B	A x B x C
4	5	5	8.00	20	160
3	10	5	8.40	15	126
2	15	5	8.50	10	85
1	20	5	8.80	5	44
Total				50	415

Average swap rate: $415 \div 50 = 8.30\%$

Again, one can make the calculation more accurate using an IRR approach, though what complicates matters is that whereas with an amortising swap only the initial “payment” was negative, for the accreting swaps, one must account for future notional payments for the theoretical bonds. Thus one “pays” \$5mm at the beginning, another \$5mm in one year, though this is partly offset by a coupon payment, and so on.

Continuing on with the accreting swap above:

Cash flows \$,000

	4-year	3-year	2-year	1-year	Total
Year 0	-\$5,000				-\$5,000
Year 1	\$400	-\$5,000		-\$4,600	
Year 2	\$400	\$420	-\$5,000		-\$4,180
Year 3	\$400	\$420	\$425	-\$5,000	-\$3,755
Year 4	\$5,400	\$5,420	\$5,425	\$5,440	\$21,685

The IRR of these cash flows is 8.29%.

The use of accreting swaps

Almost no-one knows the rationale for an accreting swap.

However, consider as an example the case of a company engaged on an engineering project which will require them to borrow regularly over the next few years but with the intention of repaying everything on completion and sale of the project.

If they want a fixed rate liability (e.g. for budgeting purposes), and they want to lock in fixed rates today through a series of forward starting loans, they will have to pay a relatively high rate as the exposure that forms the basis of the rates quoted (relative to the risk-free rate) begins not from the start date of each loan but from the date of commitment to that loan.

Consider a company who is assessed by a lender as having a credit risk worth 1% over the risk-free rate p.a. Now suppose the company wants to borrow for 1 year in 4 years' time, and the 4-year forward 1-year risk-free rate is 10%. The rate that the lender should charge will be rationally 15%, 1% a year for 5 years of exposure. For note, if the company's credit quality declines to virtual default in 4 years. The lender will still (usually) be contractually obliged to lend the money then.

Instead, suppose that the engineering company can usually borrow at LIBOR + 0.50%. Now by putting on an accreting swap paying (say) 9% versus 6-month LIBOR, the company can borrow floating rate debt each year, and the swap will have the effect of converting the floating liability to a fixed liability at 9.50%.

The company is taking a financing/basis risk. It is possible that when it comes to borrow each year it is actually quoted LIBOR + 0.75%, equivalent to a 9.75% cost. However, if the company is willing to take that risk, then the accreting swap allows the forward hedge without the expense of the credit cost.

EXOTIC SWAPS

At root, all swaps are exchanges of cash, and the amount of cash to be exchanged is determined by (the performance of) a particular asset class or index. The difficulty in pricing a swap lies in the fact that theoretically the NPV of any swap should be zero (allowing for transaction costs, fees, &c.).

For most exotic swaps, the future cash flows are not predictable, and the firm pricing the swap must attempt to find some way to impose predictability on these flows, in a way analogous to options pricing, take the uncertainty/risk itself, or broker the swap to another counterparty. The first two choices will result in a swap rate which has these uncertainties or risks priced in, as far as practical.

A good example of the “problem” is provided by the basic fixed-floating interest rate swap. Theoretically, the way in which rates are priced forward means that if you arrange today to lock in the future LIBOR payments, the PV should equal the PV of the fixed payments (else there would be risk-free arbitrage. However, in many markets it is impractical to hedge the fixed part of a swap using a series of forward LIBOR flows because there is not sufficient liquidity.

Fortunately, the vanilla swap market readily allows hedging by the simple expedient of entering into another swap with somebody else. In the exotic swaps market, it is seldom or never possible to find a perfect counterparty.

Swap chaining

It is possible to chain a series of swaps together to generate a swap of LIBOR against another index or asset. Accordingly, one can use dollar LIBOR as a universal translator. If two completely different, unrelated asset classes can both be swapped against LIBOR, you can “translate” one asset into another simply by chaining the two swaps. If you can quote a swap rate for a German company’s dividend vs. Bunds, and a swap rate for kerosene against gold, then you can construct a chain as follows: swap Bunds for DMLIBOR (standard interest

rate swap); swap EURIBOR for dollar LIBOR (standard floating-floating currency swap); swap dollar LIBOR for gold (standard commodity swap). Instead of having to think the chain through, you can simply look at the (derived) dividend swap vs. dollar LIBOR, and kerosene vs. dollar LIBOR, and it is easy enough to combine the two (although why one would want to do so is another matter...).

NOTE: many of the terms used to describe swaps are used by different market participants for different swap structures. A swap called by one trader a spread swap may be called by a second a basis swap, which will be understood by a third trader to describe something that the first trader would call an index swap.

Basis swap

A swap between two floating indices, such as 3-month vs. 6-month LIBOR, or 3-month LIBOR vs. 3-month T-bills. Used to eliminate funding mismatches, or to lock in a funding advantage present in one market while using a funding rate from another market.

Sometimes these are referred to as “index swaps”.

Where the swap is essentially between two periods of the same base, e.g., a 3 vs. 6 LIBOR swap, there should be (almost) no spread. If there were a spread, then through judicious use of deposits one could realise that spread as profit.

Example

Some optimist (O) is prepared to pay 3-m LIBOR +25 vs. receiving 6-month LIBOR for 6 months for \$10mm. Currently, 3-month LIBOR is 4%, and 6-month LIBOR is 5%. He reckons that if rates are unchanged, he will earn roughly 0.75% for 6 months.

C now borrows \$10mm at LIBOR for 3 months and lends for 6 months.

3 months pass. 3-month LIBOR is still 4%.

C receives from O: $(4\% + 0.25\%) \times 3/12 \times \$10\text{mm} = \$106,250$.

C has to repay/refinance: $(1 + .04 \times 3/12) \times \10mm
 $= \$10,100,000$

Net borrowed for next 3 months: $\$10,100,000 - \$106,250$
 $= \$9,993,750$

3 months later.

C receives from deposit: $(1 + .05 \times 6/12) \times \10mm
 $= \$10,250,000$

C repays: $(1 + .04 \times 3/12) \times \$9,993,750$
 $= \$10,093,700$

C receives from O: $(4\% + 0.25\%) \times 3/12 \times \10mm
 $= \$106,250$

C pays O: $5\% \times 6/12 \times \$10\text{mm} = \$250,000$

Net profit: $\$10,250,000 + \$106,250 - \$10,093,700 - \$250,000$
 $= \$12,550$

Annualised % profit: $\$12,550 \div \$10\text{mm} \times 12/6 \times 100\%$
 $= 0.25\%$,

equal to the margin paid by O.

The \$12,550 is the profit for a risk-free transaction, which as we know is “not allowed”. In practice, of course, bid/offer spreads, &c. may make the realisation of this profit difficult, so a small margin may be added to one side of the swap without a practical arbitrage opportunity ensuing.

In the case of a swap between T-bills and LIBOR, the arbitrage-free margin over T-bills will be determined by the (average) difference between T-bill and eurodollar futures contracts. If over the next year the difference between contracts is roughly 60 b.p., then the likely swap rate would be 3-month LIBOR vs. T-bills + 60, with a little plus or minus to allow for a bid/offer spread.

Yield curve swaps

A form of basis swap where the indices are bond yields, such as the 5-year UST yield vs. the 10-year government bond yield: you are in effect swapping the steepness of the yield curve from 5 to 10 years. Yield curve swaps can be used to hedge mismatched positions. However, there is an additional element of risk in yield curve swaps as the duration of the bonds underlying the swap are different from the indices, and position losses owing to large changes in yields may not be offset adequately by the swap profit. (Short term indices, of course, have durations roughly equal to their terms.)

Index swap

Used either to mean a basis swap or an equity swap.

Differential swaps

The commonest meaning of a differential (“diff”) swap is a cross-currency basis (i.e. floating) swap where both flows are paid in the same currency at an exchange rate fixed in advance. (Fixed currency swaps with payments in a single currency are occasionally referred to as diff swaps as well.)

The relationship between forward currency rates and LIBOR rates in different currencies is such that the NPV of a currency swap should be zero when the two LIBOR rates are exchanged with no extra margin over one LIBOR rate: if one currency has a LIBOR rate of 3%, and the other, 13%, the difference should be made up by the forward price *of the coupons* as well as on the principal. In order for the PV of a diff swap to be zero, there must be a margin to compensate for this difference.

Quanto swap

A currency swap with a variable principal (redemption) amount dependent on the value of an underlying asset, the price of which could change by the time the swap expires. Thus any forward contract used to hedge the other currency will be for an uncertain principal amount. Traditional forward contract

will not permit this, so there are alternative strategies. One alternative is to enter into a forward contract for a fixed amount and take a risk on the change in asset value with resultant currency risk. The other alternative is to incorporate into the price quoted for the spot rate through the life of the swap an extra cost that reflects the principal uncertainty, which price thus represents an embedded option.

A quanto swap can be used to put on a cross-currency asset swap where the maturity of the swap is shorter than the maturity of the asset. (There is of course no reason why you have to restrict quanto-type swaps to currency-orientated swaps.) Some market participants use the term "quanto swap" for a diff swap.

Hedging exotic swaps

The futures and forward contracts underlying the non-benchmark side of an exotic swap tend to be illiquid beyond a year or two. However, many of these exotics do not need to be hedged, for three main reasons. First, many swaps are brokered; second, they may be used to take a market view, and no hedging is desired; and third, they may themselves have been used to hedge existing positions.

EXERCISE:

SWAPS

Kraken Bank

Cachalot Industries and Pequod Corp. are clients of Kraken Bank. Cachalot want to borrow fixed funds for 10 years, and would normally issue bonds at 9%. Pequod want to borrow floating funds for 10 years, and they would normally issue an FRN at LIBOR plus $\frac{1}{4}$.

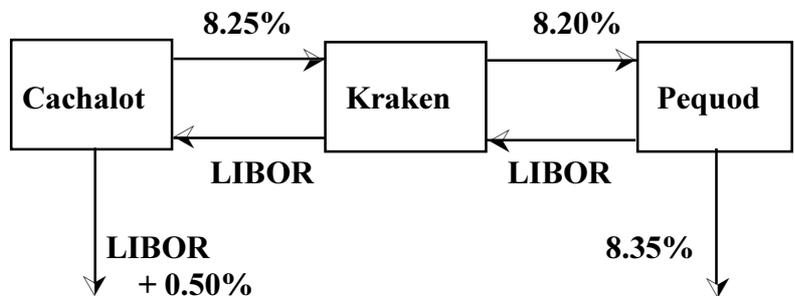
Kraken have guaranteed that Cachalot can issue an FRN at LIBOR plus $\frac{1}{2}$, and this will, through a swap, result in a saving of $\frac{1}{4}\%$ p.a. versus a fixed rate issue. Pequod are looking to save 10 cents p.a. through a swap.

If Kraken want to arrange a swap between Cachalot and Pequod, and take out 5 cents per annum in arrangement fees, what rate will Pequod have to pay on a fixed rate bond for the swap to work?

(Ignore daycounts, compounding, &c.)

Kraken Bank: solution

The easiest way to solve this is to set out the boxes and arrows in the usual fashion, putting the floating arrows at LIBOR, and then everything flows automatically.



If you solve the problem "longhand":

Cachalot

Required net cost : 8.75%
 Received on swap: LIBOR
 Paid on FRN : LIBOR + 0.50

Net paid on swap : $8.75 - (L + 0.50) + L = 8.25\%$

Kraken take 5 cents, and pay 8.20% to Pequod.

Pequod

Required net cost : LIBOR + 0.15%
 Received on swap : 8.20%
 Paid on swap : LIBOR

Therefore fixed rate paid to achieve net cost:
 $8.20 + (L + 0.15) - L = 8.35\%$

EXERCISE:

SWAPS

Elephant and Castle

Elephant, a AA bank, wants to raise 10-year fixed rate dollars. Castle, a AA corporate, wants to raise 10-year floating dollars. Both have asked Borough Bank to arrange the funding.

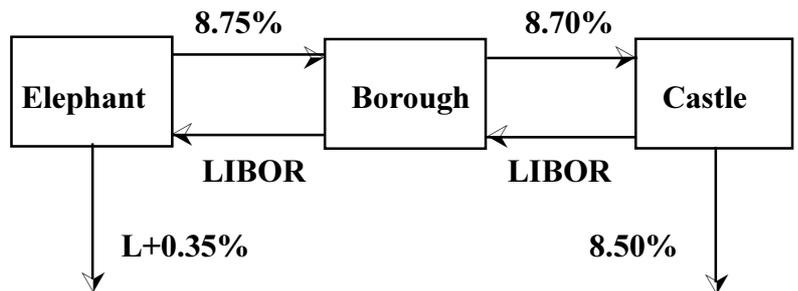
Market rates	annual fixed	floating
10-year AA corporates	8.50%	LIBOR flat
10-year AA banks	9.20%	LIBOR + 35 b.p.

Structure a swap to produce the necessary funding, allowing Borough Bank 5 b.p. p.a. as intermediary, and producing a cost of funds to Castle of LIBOR - 20 b.p.

What is Elephant's saving ?

(Ignore day-count effects.)

Elephant and Castle: solution



Elephant's cost of funds is therefore $(8.75\% + 0.35\%)$, 9.10%, a saving of 10 b.p.

EXERCISE:

SWAPS

Hercules Securities

Hercules entered into a 3-year \$10 million interest rate swap with Nemea Bank a year ago, receiving 8.00% semi-annual fixed against 6-month LIBOR. They want to unwind the swap. When calculating a tear-up fee for unwinding a swap, Nemea discount the fixed cash flows at the swap rate for the period. How much will it cost Hercules today to unwind the swap ?

UST 2-year note: 8.00%

2-year swap spreads: 40-50 b.p.

Hercules Securities: solution

1. Calculating the swap rate

Hercules have to pay the fixed part of the swap to unwind:

$$8.00\% + 50 \text{ b.p.} = 8.50\%$$

2. Calculating the present value of the swap

Hercules would pay 8.50%, versus the 8.00% they are currently receiving, a difference of 0.5% p.a., 0.25% per 6 months.

$$\text{PV of } 0.25\%/s\text{-a. for 2 years @ } 8.5\% \text{ s.a.: } 0.902$$

Dollar value of tear-up fee:

$$0.902\% \times \$10,000,000 = \$90,200$$

So Hercules would pay \$90,200 to unwind the swap.

Alternatively:

$$\text{price of 2-year 8\% bond to yield 8.50\%} = 99.098$$

$$\text{loss/unwinding cost: } (100 - 99.098) \times \$10\text{mm} = \$90,200$$

EXERCISE:

THE USE OF INTEREST RATE SWAPS

Staunton Investments

Staunton Investments will be receiving redemption proceeds from bond in the next few months, and they intend to put the proceeds received into the dollar bond market.

Staunton think that interest rates in the US will fall over the next few months, and they want to take advantage of present yield levels. You are a salesperson covering them, and you suggested interest rate swaps as a way to do this. You convinced them, and they now want indications of rates.

Swap rates vs. 3-month and 6-month LIBOR

1 year : 35-30 b.p.
3 years : 40-35 b.p.
10 years : 40-35 b.p.

U.S. Treasury yields

1 year : 5.00%
3 years : 5.50%
10 years : 6.25%

LIBOR

3-month: 4.25%
6-month: 4.25%

What swap would you suggest ?

At what rate would Staunton deal ?

Are the LIBOR rates relevant in this case ?

Staunton Investments: solution

Staunton want to hedge against falling interest rates, so they would want to receive fixed on a swap. Longer duration instruments should outperform shorter dated instruments when yields fall, so Staunton should use the 10-year swap.

When choosing between 3-month or 6-month LIBOR, the fixed receiver is in the same position as a trader locking up term funding on a position. As Staunton think that rates will fall, there is an argument in favour of them paying 3-month floating, not 6-month.

Swap rate: $6.25\% + 0.35\% = 6.60\%$

The LIBOR rates are not really relevant, as they make only a small contribution to the expected outcome.

EXERCISE:

SWAPS HEDGING

Virtual Shrubbery, Inc.

Virtual Shrubbery have taken to using interest swaps as a source of trading profits ever since internet landscape gardening became a low-margin business. They are currently receiving fixed on a pair of swaps which they wish to hedge not by unwinding but by paying fixed on a swap of intermediate maturity, as they wish to bet on the yield curve, though not interest rates.

They are not concerned with short term rates: it is just the long-term exposure they wish to hedge.

What will be the notional amount of the swap ?

Current position

Receiving fixed on \$10mm, 4-year duration, mark-to-market value \$300,000 inc. accrued.

Receiving fixed on \$16mm, 7-year duration, mark-to-market value \$900,000 inc. accrued.

Proposed swap

paying fixed for 5.5 year duration (par swap)

Virtual Shrubbery, Inc.: solution

dollar duration of position

$$4 \times \$10,300,000 + 7 \times \$16,900,000 = \$159,500,000$$

notional amount of hedging swap

$$\$159,500,000 \div 5.5 = \$29,000,000$$

EXERCISE:

CURRENCY SWAPS

Ruy Lopez Sherry

You are in the corporate finance department of Greco Bank.
You have been covering Ruy Lopez for a while in an attempt to get them to issue a bond.

Their finance director proudly tells you that he was able to issue a South African Rand eurobond (see below) which he swapped into dollars, producing a net cost of dollar funds of 6-month LIBOR plus 25 b.p. when using a dollar discount rate of 8% and a Rand discount rate of 15%.

What was the Rand swap rate vs. 6-m \$ LIBOR ?

Bond issue

15% annual for 4 years at 100 all-in cost.

Ruy Lopez Sherry: solution

As the bond was issued at par, the cost of funds must equal the coupon, i.e., 15%.

So the swap rate must have been 15% vs. (\$ LIBOR plus 25 b.p.)

To determine the swap rate against LIBOR flat you must deduct the Rand equivalent of 25 b.p. semi-annual money market in dollars, which is equivalent to approx. 26 b.p. annual.

Then you need to find out the PV of these 26 b.p. discounted at the dollar discount rate, and then re-annuitise using the Rand discount rate.

PV of 0.26 p.a. for 4 years at 8%: 0.86

4-year annuity with PV of 0.86 and a rate of 15%: 0.30.

So the swap rate must have been (15.00 - 0.30), 14.70% vs. \$ 6-month LIBOR.

EXERCISE:

PRICING ASSET SWAPS

Steinway Bank S.A.

Steinway Bank are being offered two bonds for asset swapping.
What spread over LIBOR should they be able to get ?

Bonds

Chopin 6% 5-year bond at 90

Liszt 11% 5-year bond at 109

Current swap rates

8% for all maturities out to five years

Steinway Bank S.A.: solution

As the swap curve is flat, the zero curve is also flat at 8%, so one can use 8% for all calculations.

Chopin

Price at YTM 8% = 92.0

Surplus to be re-annuitised: $92 - 90 = 2.0$

Per annum value of annuity: $2.0 \div (1/1.08 + 1/1.08^2 + 1/1.08^3 + 1/1.08^4 + 1/1.08^5) = 0.50$

Liszt

Price at YTM 8% = 112.0

Surplus to be re-annuitised: $112 - 109 = 3.0$

Per annum value of annuity: $3/2 \times 0.50 = 0.75$

EXERCISE:**AMORTISING SWAPS****Tartakower Bank**

Tartakower Bank specialises in financing engineering and industrial projects. Three of their UK clients are interested in borrowing sterling to finance these projects. As a senior loan officer you have suggested to these clients alternative structures, a single maturity loan or an amortising loan. These clients are indifferent as to whether the single maturity loans are fixed or floating.

In general, Tartakower is more interested in lending floating funds, so even if your clients go for the fixed rate alternative, you will swap the loans into floating rates.

Obviously, Tartakower want to earn as high a spread over LIBOR as possible. Through your effective coverage of these clients, you know that they will go along with whichever funding alternative you recommend.

What do you recommend to each client ?

What margin over LIBOR will Tartakower receive ?

Nimrod Ziggurats

Floating rate	: 6-month LIBOR + 25 b.p. for 5 years
Fixed rate	: 7.95% annual
	Single maturity : 5 years
	Amortising : 50% in year 4, 30% in year 5, 20% in year 6

Noah Marine

Floating rate	: 6-month LIBOR + 25 b.p. for 3 years
Fixed rate	: 7.70% annual
	Single maturity : 3 years
	Amortising : 20% p.a. for 5 years

Hiram Timber

Floating rate : six-month LIBOR + 25 b.p. for 2 years
Fixed rate : 7.20% annual
Single maturity : 2 years
Amortising : 50% in year 1, 25%
p.a. in years 2 and 3

Sterling swap rates (annual) vs. 6-month LIBOR

1 year : 6.80 - 6.70%
2 years : 7.00 - 6.90%
3 years : 7.40 - 7.30%
4 years : 7.50 - 7.40%
5 years : 7.70 - 7.60%
6 years : 7.75 - 7.60%

Tartakower Bank: solution

The target to beat in all cases is the LIBOR plus 25 b.p. on the floating rate loans. (No account need be taken of day-counts, as sterling money market is A/365. You must, however, convert the annual margin in b.p.s on the fixed part of any swap into the semi-annual equivalent.

1. Nimrod Ziggurats

Single maturity loan

The fixed rate of 7.95% can be swapped into floating at LIBOR plus 25 b.p. 25 b.p. annual is less than 25 b.p. semi-annual, so the floating rate loan will have the higher spread.

Amortising loan

Average swap rate

Year	Amount	Rate	Weight	Weight
A	B	C	A x B	A x B x C
4	50%	7.50%	200	1,500
5	30%	7.70%	150	1,155
6	20%	7.75%	120	930
Total			470	3,585

Average swap rate: $3,585 \div 470 = 7.628\%$

The amortising loan, if swapped, generates LIBOR plus 32.2 b.p. annual, roughly LIBOR plus 31 b.p. semi-annual.

So you would recommend that Nimrod accept a fixed rate amortising loan and Tartakower receive LIBOR plus 31 b.p. on the swap.

2. Noah Marine

Single maturity loan

Swapping the single maturity loan produces a floating rate of LIBOR plus 30 b.p. as the swap rate is 7.40%. 30 b.p. annual is roughly 29 b.p. semi-annual equivalent.

Amortising loan

Average swap rate

Year	Amount	Rate	Weight	Weight
A	B	C	A x B	A x B x C
1	20%	6.80%	20	136
2	20%	7.00%	40	280
3	20%	7.40%	60	444
4	20%	7.50%	80	600
5	20%	7.70%	100	770
Total			300	2,230

Average swap rate: $2,230 \div 300 = 7.433\%$

So the amortising loan, if swapped, would produce a floating rate of LIBOR plus 26.7 b.p. (annual).

You would therefore recommend the single maturity fixed rate loan and Tartakower would get LIBOR plus 29 b.p. on the swap.

3. Hiram Timber

Single maturity loan

The fixed rate single maturity loan at 7.20% can therefore be swapped into LIBOR plus 20 b.p. as the swap rate is 7.00%. Clearly, the fixed loan cannot be swapped to exceed the LIBOR plus 25 b.p. on the floating rate loan.

Amortising loan

Average swap rate

Year	Amount	Rate	Weight	Weight
AB	C	A x B	A x B x C	
1	50%	6.80%	50	340
2	25%	7.00%	50	350
3	25%	7.40%	75	555
Total			175	1,245

Average swap rate: $1,245 \div 175 = 7.114\%$

Clearly, the amortising swap is even less favourable to Tartakower.

So Hiram should take the floating rate loan and Tartakower would receive an unexciting LIBOR plus 25 b.p.

EXERCISE:

EXOTIC SWAPS

Renfield Bank

You are in charge of new swaps products at Renfield Bank, and, as such, you are occasionally called on to design swaps to meet specific client requirements.

You have people working for you who run the numbers, so in general all you have to do is conceptualise the swap structures and let your whizz-kids do the rest. What structures will you devise ?

You will have to be prepared to comment on these structures, e.g., hedging, problems in structuring, &c.

1. Lambing Oil, an American oil production company who are concerned over the effect that pollution controls will have on refining costs.

Current oil prices

WTI : \$15/barrel.

Refined oils: \$16/barrel (basket value of oils produced).

2. Brumzim plc, a top UK company and a barometer of UK corporate health. They believe that their profitability will be adversely, though predictably, affected if swap spreads widen over the next two years, as this will raise fixed and might raise floating rate costs.

Current rates

2-year gilts: 7.00%

Swap rates : 65-60 b.p.

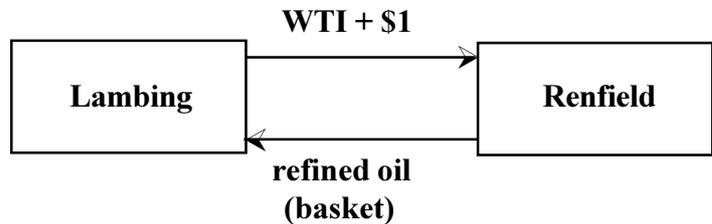
3. The Really Greasy Group plc, a specialist caterer. They provide a pre-packed frozen English breakfast to service stations and small hotels all around the UK.

They have unearthed a strong positive correlation between demand for their product and the strength of the UK stock market.

4. Paisano & Bauer is a German/Italian network of low-budget agricultural producers who believe that a major recession in the UK will tend to increase demand for their staple foods.

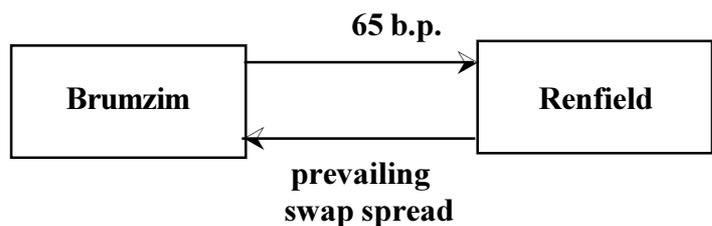
Renfield Bank: solution

1. If Lambing are right, then the crack spread (crude versus refined oil) will widen. Accordingly, a suitable swap might look like this:



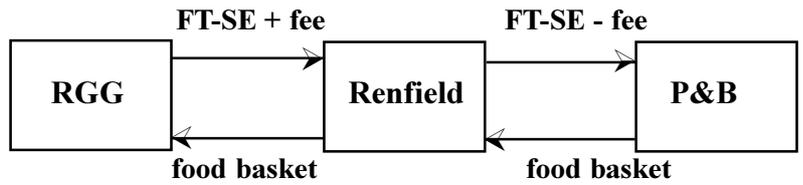
This swap can be hedged with futures as far out as possible. In the longer term, Renfield will have an exposure, although the exposure can be reduced by rolling the futures as far contracts get nearer and more liquid. Renfield should be able to charge a high fee as part of the exposure will not be properly hedged.

2. A possible swap could be:



This swap should probably be hedged by finding a suitable counterparty. If Renfield itself pays fixed on an interest rate swap and hedges with gilts or gilt futures, thus eliminating the interest rate component, they are still locked into paying a credit spread unnecessarily.

3. and 4. By structuring the swap carefully, you should be able to combine the two requirements in the same swap and collect a fee from both clients.



The problems arise through the composition of the commodity basket, the ratio between that basket and the FT-SE 100, and the currency of settlement (although sterling seems reasonable).